ANALYSIS OF SERIES DIPOLE COMPENSATION

BY ROOT LOCUS TECHNIQUES

By

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PREFACE

Since the introduction of the root locus method by Evans in 1950, it has enjoyed an ever-increasing popularity for the analysis and design of feedback control systems. Today, the widespread teaching and application of the root locus technique leave little to distinguish it from a truly classical procedure.

For purposes of design, the root locus method exhibits a significant advantage. Information concerning the system performance is furnished in a form amenable to corrective action by the designer to improve the performance. That is, it is possible to interpret required over-all performance characteristics in terms of individual system elements.

Frequently, acceptable performance is derived through the use of series compensation of the system. A dipole compensator of the phase lead or phase lag type when applied to a system can be expected to alter the characteristics of the system and these changes in performance, and their extent, may be gleaned from the plot of the root locus of the system. In this manner, suitable compensation may be achieved.

Several techniques involving the root locus method are valuable in displaying information about the system which serves series compensator design. This thesis is primarily concerned with the investigation and further development of these techniques.

The author is pleased to acknowledge the guidance and encouragement of Professor Paul A. McCollum during the preparation of this work. Also, to my sister, Lois, who gallantly accepted the chores of typing, go my heartfelt thanks.

Lastly, I note that this thesis, indeed my entire graduate program, would not have been possible without the patience and understanding of my wife, Jeanne.

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CHAPTER I

INTRODUCTION

Current design procedures for feedback control systems are applied in two general directions of attack. First, to design a completely new system to meet given performance requirements; or second, given the system requirements and an existing unsatisfactory system, to design compensating devices which allow the existing system, with compensation, to deliver acceptable performance. The second category is usually most familiar to the control systems designer since many original-design systems must be further compensated to achieve desired performance characteristics.

The over-all purpose of compensation is to improve the performance of a system so that its properties approach those of an ideal control device. An ideal or perfect feedback control system would be capable of producing an output signal or value of the controlled variable which is always in exact correspondence with the desired value represented in the input signal to the system. The perfect control system is naturally elusive and in a physical form remains out of reach of the designer.

Unfortunately, control systems emerge as a collection of physical components with inherent properties of attenuation and time delay. Were it not for these phenomena, the ideal feedback control system would be within the grasp of every systems designer. By reason of this departure of the physically realizable system from the ideal or perfect system;

the specifications for any given system design are somewhat complicated.

To prescribe the allowable deviation of the physical system from the ideal, necessary design criteria have been established. These criteria describe the system performance in terms of over-all stability, system gain, and transient and steady-state operation. Naturally, not every one of these specifications can be arbitrarily fixed for a given system, since improvement in one area of system performance may necessitate the compromise of desirable characteristics in another.

This give-and-take atmosphere certainly does not ease the problems of the systems designer, but rather it shouts the need for a method of analysis and design which yields a clear picture of the interaction between system gain, stability, and transient and steady-state performance.

Several distinct methods of analysis and design are available to the control engineer today, each method possessing some advantage over the others depending on the type of information desired or known about the system.

Figure I-1 represents the unity feedback control system in block diagram form.

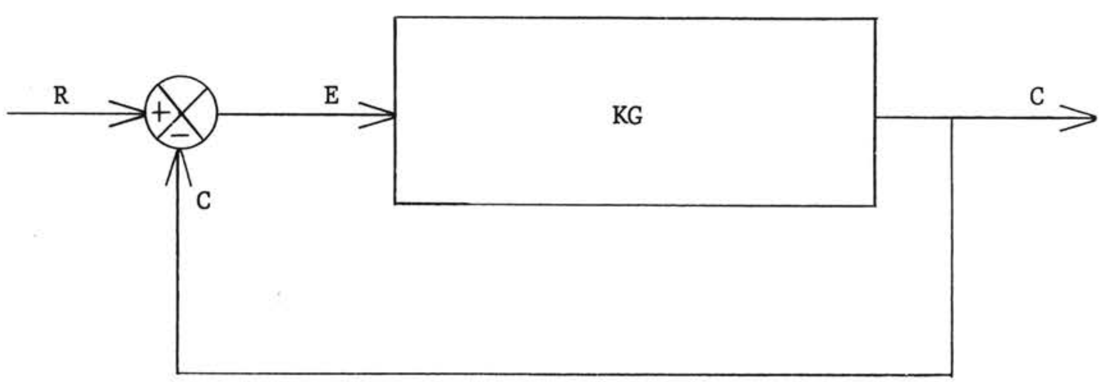


Figure I-1. Unity Feedback Control System

KG represents the product of K, the system gain constant, and G, the transfer function of the system components. R is the reference input signal, C is the output or controlled variable, and E is the error signal defined as E = R - C. Since C = KGE, then

$$\frac{C}{R} = \frac{KG}{1 + KG} \tag{I-1}$$

Equation (I-1) describes the ratio of the controlled variable to the reference in terms of the system transfer function for unity feedback.

The early analysis and design methods were based upon the solution of the linear differential equation of the system. (1,2). The equation is given in terms of the component parameters such as inertia, friction, capacitance, inductance, etc., and is normally of second or higher order. The classical methods of solution of linear differential equations yield the time response of the system for specific boundary conditions.

This method yields an exact expression for the output, C, of the system in Figure I-1 for a given input signal R, and in so doing, describes the transient response of the system. However, if the response so obtained is not acceptable in the light of the desired characteristics of the system, it is difficult to determine from the differential equation alone which system parameters should be altered (and to what extent) to improve the system performance. This inherent difficulty limits the usefulness of the transient response method in design procedures.

The need for simpler and more rapid design procedures was somewhat alleviated with the advent of the transfer function method of analysis (attenuation and phase plots). This approach allows the over-all system to be sub-divided into individual elements, each with its own mathematical expression describing its performance. Although the information obtained is derived in general by graphical methods, it is possible to determine the contribution of the individual elements to the total system performance. This property endows the transfer function method with a striking advantage over the transient method. When the system performance is shown to be unsatisfactory by transfer function analysis, the designer is furnished with a perceptive insight as to which parameters in the system must be altered to provide acceptable operation.

The transfer function approach, although fulfilling the role of a much improved design method, is not as rigorous in furnishing the exact transient response. Therefore, the transient method is normally required to prove the design.

Evans (3,4,5) skillfully combined the desirable features of the transient response method with the transfer function technique in his root locus method. In a fashion similar to the transfer function approach, the root locus method is graphical in nature and the plot is obtained from the transfer function expression of the system. In addition, the system performance is furnished in terms of the contributions of the individual elements in the system, thus aiding in design. As a further advantage, the root locus method yields the roots of the characteristic equation of the system, thus providing transient response data.

Perhaps the greatest handicap in applying the root locus method has been the lack of understanding and appreciation of pole-zero concepts on the part of the practicing design engineer. However, a review of current literature indicates that the root locus is being established as a valuable design tool for the synthesis of feedback control systems. For this reason, the investigation described in this paper is based upon the root locus method. A further discussion of root locus together with specific rules for developing the plot will be advanced in Chapter II.

Let us return now to the problem which confronts the systems designer. It may be required that an existing feedback control system be compensated in some manner to improve the over-all system performance. In this case, the designer must evaluate the characteristics of the existing system and then determine the most satisfactory compensating network or device which will produce an acceptable composite system. To this end, a clear understanding of the effect of compensating devices on a given system is of paramount importance.

Many systems which possess unsuitable performance characteristics may be satisfactorily compensated through the addition of a series phase lead or phase lag network. Such a compensation scheme is shown in Figure I-2.

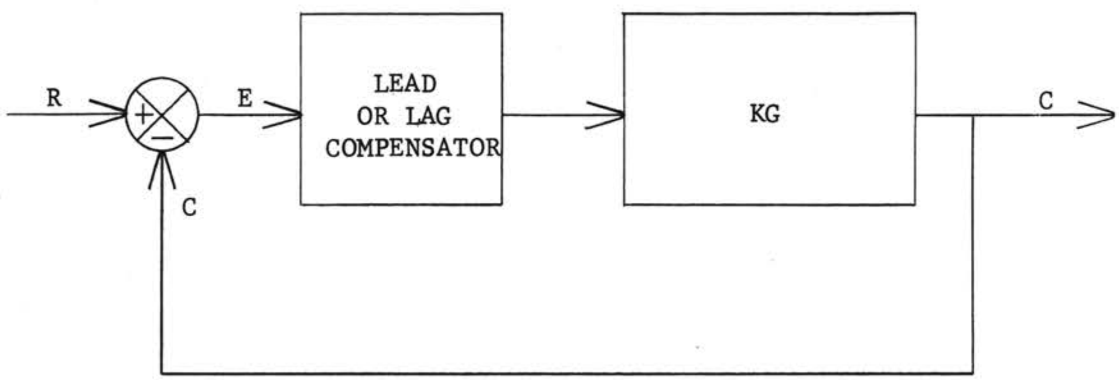


Figure I-2. Unity Feedback Control System with Series Compensation

Recent work by Ross, Warren and Thaler (6) has shown the ease with which the root locus concept can be applied to the determination of series pole-zero compensation networks. Their approach is to select arbitrarily the location of a desirable closed-loop root. This root generally will not lie on the locus of the uncompensated system. In order to force the locus of the compensated system to pass through the selected root location, the dipole compensator must contribute the proper angle increment to satisfy the fundamental root locus condition of $n \cdot 180^{\circ}$ (n = any odd integer). An infinite number of dipole compensators are capable of satisfying only the phase angle requirement. However, if the steady-state error is specified by prescribed values of K_p, K_v, K_a, etc., then the pole and zero of the dipole are fixed in order to produce the prescribed gain at the desired root location.

The method outlined by Ross ____ etc., is fast and completely analytic after the locus of the uncompensated system is obtained and the position of the relocated closed-loop root is selected. But several limitations are apparent. First, the selection of the relocated root at a specified system gain does not allow for the possibility that a more satisfactory root might be produced upon the further variation of a system parameter. Second, and probably more important, no information about the location of the remaining closed-loop roots is furnished at the time the relocated root is selected.

To facilitate the judicious selection of the parameter values for a series compensator, the over-all effects caused by variation of compensator parameters must be known. This infers that a knowledge of the movement or shifting in the complex plane of all of the closed-loop roots versus compensator parameter variation is required. The investigation described in this thesis concerns techniques for deriving this information from the root locus plot.

CHAPTER II

THE ROOT LOCUS METHOD

Introduction

The primary purpose of this chapter is to discuss, in general, the construction of the root locus and to examine the procedures for obtaining system performance data from the root locus plot.

Of secondary importance is the writer's desire to preserve continuity and to introduce notational forms regarding the root locus which will serve to facilitate the discussion in the later chapters.

The following discussion is similar in content to sections dealing with root locus in most recent texts on feedback control system analysis and design. (7,8,9). Its lack of rigor and detail presumes some knowledge of the root locus technique on the part of the reader.

General Development of the Root Locus

Rewriting Equation (I-1) as a function of the complex frequency, (s), it becomes

$$\frac{C}{R}(s) = \frac{KG(s)}{1 + KG(s)} \tag{II-1}$$

which represents the closed-loop function of a system with unity feedback. KG(s) is the open-loop transfer function and normally is expressed as the ratio of two rational polynomials in (s) and the multiplicative gain constant, K

$$KG(s) = \frac{KN(s)}{D(s)} \tag{II-2}$$

Then Equation (II-1) can be rewritten as

$$\frac{C}{R}(s) = \frac{KN(s)/D(s)}{1 + KN(s)/D(s)}$$

$$= \frac{KN(s)}{D(s) + KN(s)} \tag{II-3}$$

Equating the denominator of Equation (II-3) to zero yields the characteristic equation of the system

$$D(s) + KN(s) = 0 \tag{II-4}$$

The roots of Equation (II-4) are called the closed-loop roots of the system and they define its transient response. The root locus provides a graphical method for determining these closed-loop roots.

Roots of Equation (II-4) also satisfy the relationship

$$KG(s) = -1 \tag{II-5}$$

Since KG(s) is complex in form and thus can be expressed in terms of magnitude and phase angle, Equation (II-5) can be separated into the more basic conditions

$$\underline{/G(s)} = \underline{/n \cdot 180^{\circ}} \tag{II-6}$$

and $$|KG(s)| = 1 \tag{II-7}$$

when (n) is any odd integer.

The actual root locus plot is the locus of all values of (s) on the complex plane which satisfy the phase angle condition expressed by

Equation (II-6). Unique values of (s) become the closed-loop roots upon specification of K in Equation (II-7).

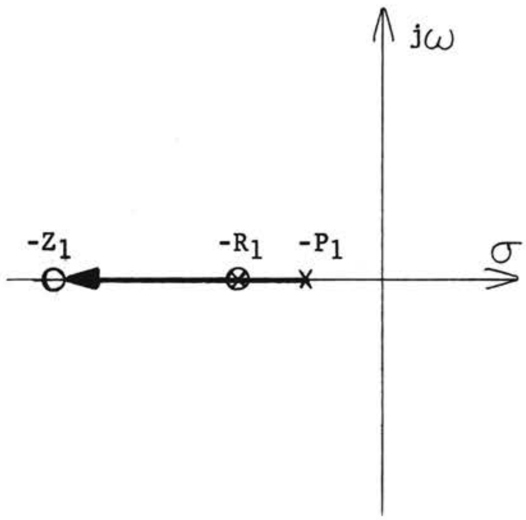
Examining Equation (II-2) more closely, the values of (s) which make N(s) zero are called zeros of N(s) and are obviously zeros of the open-loop function KG(s) and also of the closed-loop function $\frac{C}{R}$ (s). The values of (s) which make D(s) zero are called the zeros of D(s) and are referred to as the poles of KG(s) in that these values of (s) cause KG(s) to become infinite.

Equation (II-7) forces the closed-loop roots to vary with the gain. That is, if $|KG(s)| = 1$, then, at the poles of KG(s), K = 0 and at the zeros of KG(s), K = ∞ . For some finite non-zero value of K, the closed-loop roots will assume intermediate positions on the locus between the open-loop poles and the zeros. Then the root locus plot of the open-loop transfer function, KG(s), describes the position on the s-plane which the closed-loop roots [zeros of (1 + KG(s)] occupy for values of the system gain, K, which may range from zero to infinity.

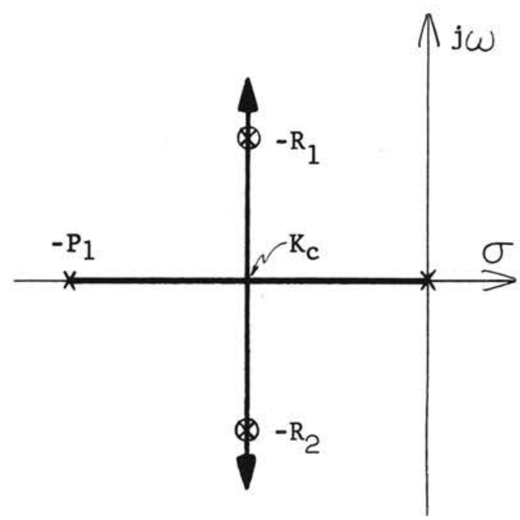
A clearer understanding of these relationships is best served with several generalized examples shown in Figure II-1.

Figure II-1(a) shows the plot of a simple dipole network. Only one open-loop pole, P_1, exists, therefore there is only one branch of the root locus and it terminates at the open-loop zero, Z_1. Also, only one closed-loop root, R_1, exists and its position on the locus is due to some finite value of gain, K. For this particular system, no value of gain will produce an oscillatory root.

The system whose plot is shown as Figure II-1(b) will present oscillatory characteristics if the gain is greater than K_c. Two open-loop poles cause two branches of the locus to be formed and two closed-loop



$$\text{(a)} \quad KG(s) = \frac{K(s + Z_1)}{(s + P_1)}$$



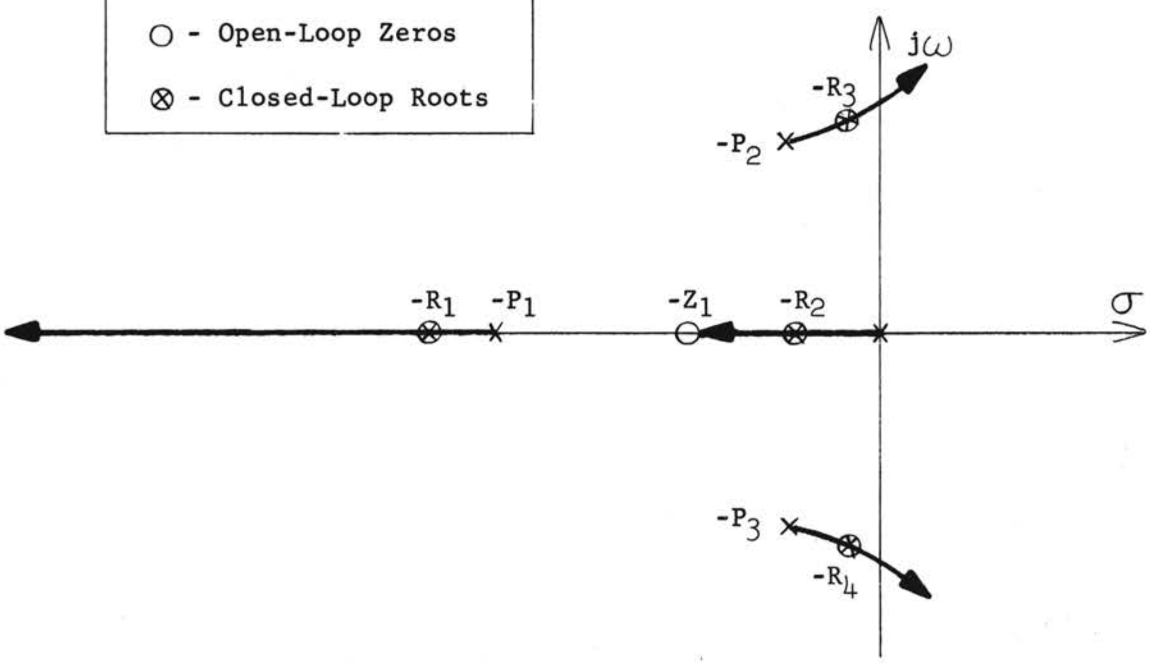
$$\text{(b)} \quad KG(s) = \frac{K}{s(s + P_1)}$$

NOTE:

× - Open-Loop Poles

○ - Open-Loop Zeros

⊗ - Closed-Loop Roots



$$\text{(c)} \quad KG(s) = \frac{K(s + Z_1)}{s(s + P_1)(s + P_2)(s + P_3)} \quad ; \; P_1 \text{ and } P_2 \text{ complex.}$$

Figure II-1. Generalized Examples of Root Locus Plots.

roots are obtained. In this case the closed-loop roots approach zeros at infinity as $K \longrightarrow \infty$.

Figure II-1(c) is the root locus plot of a system with one open-loop zero and four open-loop poles. This requires four branches of the root locus and four closed-loop roots. Two of the roots will always remain on the negative real axis for positive values of gain, and two will always be oscillatory. The location of the four roots is shown for a comparatively low value of system gain. For higher values of gain, the system becomes unstable due to the movement of roots R_3 and R_4 into the right-half s-plane. More will be said later in this chapter concerning system stability.

Obviously, an unlimited number of open-loop pole-zero combinations are possible with an attendant number of possible root locus configurations. However, the plotting of the root locus for any given function is almost a routine procedure after definite rules have been established.

The rules listed here which are helpful in constructing the root locus have been selected from lists presented by recent authors in the control systems field. The reader is referred to these sources for more complete descriptions of these rules, and in some cases, their proofs. (10, 11).

Rules for Constructing the Root Locus

NOTE: p and z represent the number of open-loop poles and zeros, respectively.

1. The locus has symmetry with respect to the real axis.
2. A branch of the root locus exists for each open-loop pole.
3. The closed-loop roots move from the open-loop poles at $K = 0$ to

the open-loop zeros or infinity at $K = \infty$.

4. The number of branches of the locus which extend to infinity is equal to (p - z).
5. The locus will exist along a region of the negative real axis if the sum of the poles and zeros on the axis to the right of the region is odd.
6. The angle of departure of the locus from an open-loop pole is equal to $\pm n \cdot 180^{\circ}$ minus the net angle contributed by vectors drawn from the remaining poles and zeros to the pole under consideration, where (n) is any odd integer. The angle of at an open-loop zero can similarly be found.
7. The locus which approaches infinity does so along an asymptote. The angle between the asymptote and the real axis is given by $\pm n \cdot 180^{\circ}/(p - z)$.
8. The intersection of the asymptotes with the real axis is given by:

$$\frac{\sum \text{real parts of poles} - \sum \text{real parts of zeros}}{p - z}$$

9. If only real open-loop poles and zeros exist, the point at which the locus leaves the real axis is given by equating the reciprocals of the distances from the poles and the zeros on the real axis to zero. A more involved procedure is required when complex open-loop poles or zeros are present. (12).

The main advantages of the root locus method are realized only when the locus itself can be quickly sketched and the closed-loop roots determined with a minimum of time and effort.

The use of the spirule for plotting the root locus is of substantial benefit. (13). The device enables the user to add and subtract angles quickly and provides a logarithmic method of finding the products of the vector lengths to evaluate gain.

For exact determination of the location of closed-loop roots, a method for numerically producing the root locus has been developed by Donahue. (14).

Stability Considerations

A fundamental requirement of any feedback control system is stability. Regardless of the type of input or change in the reference signal, the value of the controlled variable or output of the system must eventually approach the desired condition.

The stability of a feedback control system is determined by the roots of the system's characteristic equation. These roots may be obtained directly from the root locus plot and exist in either real or complex form as previously illustrated by Figure II-1.

The closed-loop function of a unity feedback control system is repeated here for convenience.

$$\frac{C}{R}(s) = \frac{KN(s)}{D(s) + KN(s)} \tag{II-8}$$

If the numerator of Equation (II-8) is written in terms of the open-loop zeros, Z_1, Z_2, ... Z_z and the denominator in terms of the closed-loop roots, R_1, R_2, ... R_p, then

$$\frac{C}{R}(s) = \frac{K(s + Z_1)(s + Z_2) \ \ldots \ (s + Z_z)}{(s + R_1)(s + R_2) \ \ldots \ (s + R_p)} \tag{II-9}$$

where z and p are equal to the number of open-loop zeros and poles, respectively. The number of closed-loop roots or poles is always equal to the number of open-loop poles.

From Equation (II-9), C(s) becomes

$$C(s) = \frac{R(s)\ K(s + Z_1)(s + Z_2)\ \ldots\ (s + Z_z)}{(s + R_1)(s + R_2)\ \ldots\ (s + R_p)} \tag{II-10}$$

The inverse transform of Equation (II-10) will yield C(t), the transient response for a given input signal, R(t).

If any of the closed-loop roots R_1, R_2, R_3, ... R_p have positive real parts, then the time response C(t) will possess one or more divergent exponential terms and the system will be unstable. Then, for system stability, all closed-loop roots must lie in the left-half s-plane, thus contributing only decaying exponential terms in the transient response.

Referring to Figure II-1, it can be seen that the systems represented by root locus plots (a) and (b) will be stable regardless of the value of gain. The system shown in (c) will be conditionally stable. That is, so long as the closed-loop roots R_3 and R_4 reside to the left of the $j\omega$ axis, the system will be stable. If the gain were adjusted so that R_3 and R_4 were located directly on the imaginary axis, any disturbance would cause the system to break into sustained oscillation. The frequency and magnitude of such oscillations can be determined handily from the root locus plot as will be shown shortly.

System instability can be corrected in either of two ways. First, the system gain can be reduced or second, the root locus may be reshaped. The purpose of reshaping the locus is to relocate the closed-loop roots in the stable region without altering the gain. Often the second alternative is the most desirable, so that speed of response and small steady-state error will not be sacrificed.

Transient Performance

Assuming that a given feedback control system is stable; that is, all of the closed-loop roots of the system lie in the left-half s-plane, then the designer is assured that eventually the controlled variable will come into near correspondence with the desired reference. The word "eventually" is rather vague and unsatisfactory in terms of the controlling properties of a feedback system. It is naturally desirable to have the controlled variable reflect the value of the reference at all times.

However, immediately after an input disturbance to a stable system, the value of the controlled variable passes through a transient condition before approaching the desired final value. This transient condition is described by the locations of the closed-loop roots in the complex plane. Therefore, the system's transient performance depends on these locations.

It is common to evaluate the transient performance in terms of the response of the system to a step input, since this type of input represents a comparatively violent system disturbance, and a somewhat pessimistic picture of the transient response is obtained.

Assume that a system has only two closed-loop roots and they occupy positions on the negative real axis as shown in Figure II-2.

Equation (II-10) represents the Laplace transform of the time response of a system to any given input R(t). Applying the conditions

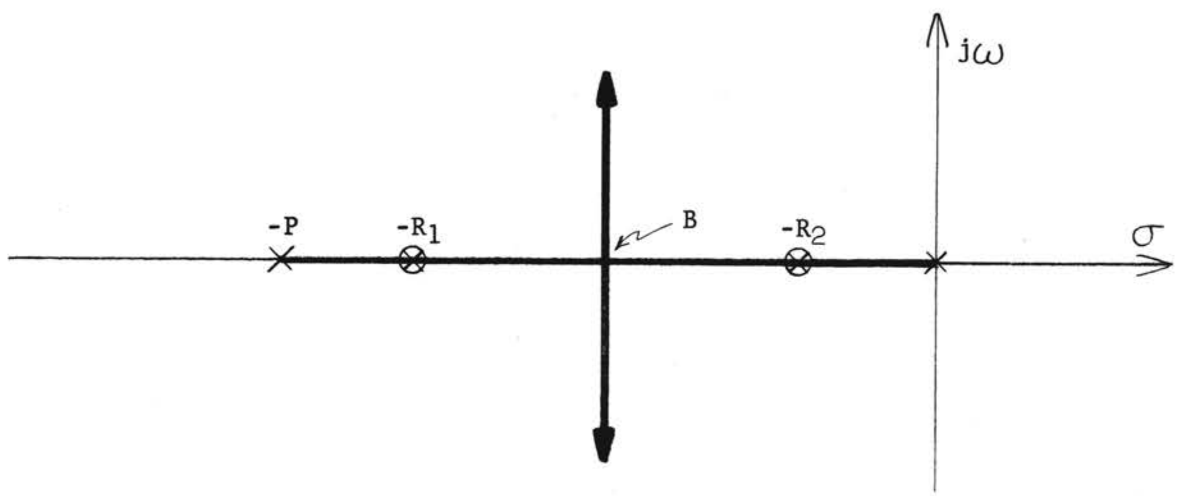


Figure II-2. Root Locus of $KG(s) = \frac{K}{s(s + P)}$

of the present example to Equation (II-10)

$$C(s) = \frac{1}{s} \cdot \frac{K}{(s + R_1)(s + R_2)} \qquad \text{(II-11)}$$

where the factor, (1/s), represents R(s) for a unit step input.

Applying the inverse transform to Equation (II-11), C(t) is found to be of the form

$$C(t) = 1 + Ae^{-R_1 t} + Be^{-R_2 t} \qquad \text{(II-12)}$$

The coefficients A and B may be determined easily from the root locus by evaluating the residue of C(s) in the roots R_1 and R_2. (15). Note that for a stable system, R_1 and R_2 are positive quantities from which decaying transient terms will be obtained.

The steady-state value of C(t) is unity. How quickly C(t) approaches this final value naturally depends upon the time constants, $1/R_1$ and $1/R_2$. For a short time constant and therefore a fast decay of the exponentials, the closed-loop roots R_1 and R_2 must be located as far away from the origin as possible on the negative real axis.

Should the gain of the system in Figure II-2 be increased, the roots R_1 and R_2 would join on the real axis at the breakaway point B, then move into the 2nd and 3rd quadrants as shown in Figure II-3.

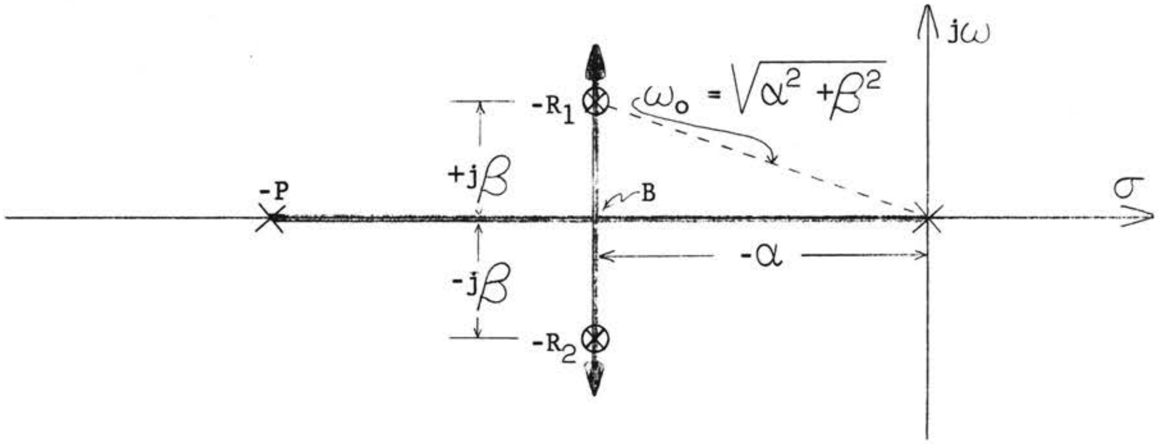


Figure II-3. Root Locus of $KG(s) = \frac{K}{s(s + P)}$ with Oscillatory Roots

The closed-loop roots are now oscillatory in nature and are of the form

$$R = \alpha \pm j\beta \tag{II-13}$$

where α is the decay constant and β is the natural frequency of oscillation associated with the complex roots. The time response to a unit step function input may be obtained by application of Equation (II-12).

$$C(t) = 1 + Ae^{(-\alpha + j\beta)t} + Be^{(-\alpha - j\beta)t} \tag{II-14}$$

where R_1 and R_2 are now complex conjugates.

Other useful relationships which describe the transient response may be derived from Figure II-3. (16). One quantity of some importance is the decrement or damping factor

$$\zeta = \frac{\text{decay constant}}{\text{undamped natural frequency}} \tag{II-15}$$

where the undamped natural frequency (ω_o) is defined as

$$\omega_o = \sqrt{\alpha^2 + \beta^2} \tag{II-16}$$

The damping factor (ζ) is very useful in that it directly furnishes information about the relative damping of the system. When (ζ) $<$ 1.0, the system is underdamped. Critical damping is indicated by (ζ) = 1.0 and overdamping results when (ζ) $>$ 1.0. Notice that critical damping occurs when the roots R_1 and R_2 of Figure II-3 reside on the negative real axis at point B.

For the case of an underdamped system, Equation (II-14) may be written in terms of the damping factor (ζ) and the undamped natural frequency (ω_o) as

$$C(t) = 1 - \frac{e^{-\zeta\omega_o t}}{\sqrt{1 - \zeta^2}} \sin\left(\omega_o \sqrt{1 - \zeta^2}\, t + \phi\right) \tag{II-17}$$

where (ϕ) is given by

$$\phi = \tan^{-1} \frac{\sqrt{1 - \zeta^2}}{\zeta} \tag{II-18}$$

Figure II-4 shows a typical time response of an underdamped second-order system with a unit step input. The time for the maximum value of the controlled variable to occur is given by

$$T_p = \frac{\pi}{\beta} \tag{II-19}$$

Equation (II-19) indicates that an increase in the frequency of

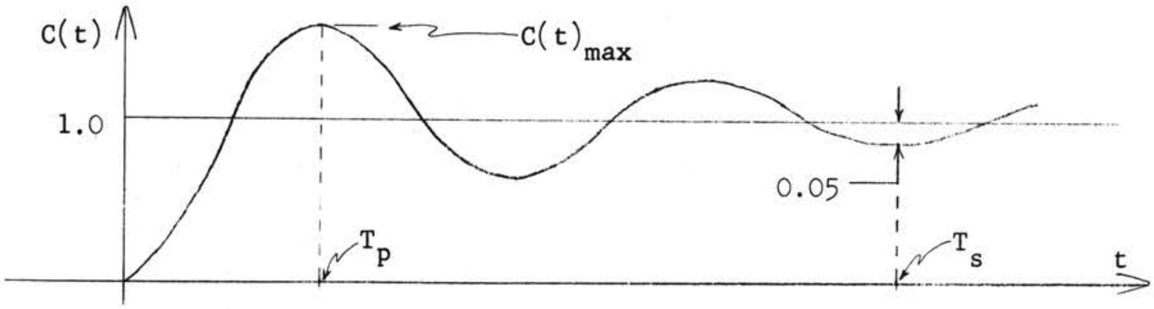


Figure II-4. Time Response of a Second-order System for a Unit-Step Input

oscillation, β, will result in a reduced T_p and, therefore, a system with faster response.

The maximum value of C(t) occurring at the time T_p may be expressed conveniently in terms of the damping factor

$$C(t)_{max} = 1 + e^{-\left(\frac{\zeta \pi}{\sqrt{1 - \zeta^2}}\right)} \tag{II-20}$$

The settling time, T_s, is that amount of time required for the controlled variable to remain within a certain per cent of its final value. If a 5% error is assumed then the setting time becomes

$$T_s = \frac{3.0}{\zeta \omega_o} \tag{II-21}$$

Although the foregoing discussion has been restricted to a system with only one pair of complex closed-loop roots, many systems produce two or more pairs of conjugate roots. Also, single roots may occur on the real axis. Naturally, the actual transient response is a culmination of the response contributions of each of the individual roots. However, if the location of a pair of complex roots is such that they dominate the total response, i.e., the roots have long time constants compared with other system roots, then the relationships given in Equations

(II-14) through (II-21) are of great help in evaluating the transient response of the system.

It should be noted that those roots which occur close to the imaginary axis have the most lasting effect on the transient response. It is the location of these roots which will be subject to close examination when the selection of a dipole compensator is made.

Steady-State Performance

A stable feedback control system having experienced a disturbance in the form of an input reference change, and having undergone the transient condition, then enters the steady-state region of operation. The steady-state mode does not infer that the input and output signals are constant. For instance, the input reference may be in terms of shaft position. The shaft position may assume a constant acceleration, constant rate of change of acceleration, etc., and steady-state operation of the system may result, depending on the system type.

The steady-state performance may be evaluated readily from the error coefficients and these coefficients, in turn, may be derived from the closed-loop poles and zeros. (17). The ratio of the error signal E(s) to the reference input signal R(s) can be expressed as

$$\frac{E}{R}(s) = C_0 + C_1 s + C_2 s^2 + \ldots \qquad \text{(II-22)}$$

where C_0, C_1, C_2, ... are the error coefficients.

For a unity feedback system, C(s) = R(s) - E(s) and Equation (II-22) may be rewritten as

$$\frac{C}{R}(s) = 1 - C_0 - C_1 s - C_2 s^2 - \ldots \qquad \text{(II-23)}$$

Repeating Equation (II-9),

$$\frac{C}{R}(s) = \frac{K(s + Z_1)(s + Z_2) \ldots (s + Z_z)}{(s + R_1)(s + R_2) \ldots (s + R_p)}$$

Manipulating Equation (II-9) into a series expansion in (s), then equating coefficients of like powers of (s) in Equation (II-23), the error coefficients, C_0, C_1, C_2, ... may be expressed in terms of K, the open-loop zeros, Z_1, Z_2 ... Z_z, and the closed-loop roots, R_1, R_2, ... R_p.

$$\left.\begin{aligned} C_0 &= 1 - \frac{K(Z_1 Z_2 Z_3 \ldots Z_z)}{R_1 R_2 R_3 \ldots R_p} \\ C_1 &= \sum_1^p \frac{1}{R_i} - \sum_1^z \frac{1}{Z_j} \\ C_2 &= -C_1 \sum_1^p \frac{1}{R_i} + \frac{1}{2}\left(\sum_1^p \frac{1}{R_i R_j} - \sum_1^z \frac{1}{Z_i Z_j}\right) \end{aligned}\right] \quad \text{(II-24)}$$

Coefficients of higher order may be derived as shown above, but normally only those of lower order are of prime importance in assessing the steady-state performance of a system. The foregoing statement is valid when the input signal contains no higher derivatives of displacement than second order. Note that the error coefficients do not reflect the transient components of the input variation, but only those components of the input signal and its derivatives which are sustained for a length of time which will allow the system to move into steady-state operation.

Once the error coefficients have been found from the root locus and Equations (II-24), the steady-state error of a system to a given input can be readily determined from Equation (II-22). Applying the final value theorem to Equation (II-22), the steady-state error becomes

$$E_{ss} = \lim_{s\to o} sE(s) = \lim_{s\to o} sR(s) \left[C_0 + C_1 s + C_2 s^2 + \ldots\right] \tag{II-25}$$

Feedback control systems are normally classified according to type. When the transfer function of the control element is written in time constant form

$$KG(s) = \frac{K(\tau_1 s+1)(\tau_2 s+1)\ldots(\tau_z s+1)}{s^N(\tau_a s+1)(\tau_b s+1)\ldots(\tau_p s+1)} \tag{II-26}$$

The exponent N of the series integration factor $1/(s)^N$ describes the system as type 0, type 1, type 2, etc., as N assumes values of 0, 1, 2 etc., respectively. K in Equation (II-26) is also given particular notation forms for the several usual system types found in practice. K becomes K_p, K_v, and K_a for types 0, 1, 2, respectively.

Certain relationships exist between the system gain constants K_p, K_v, and K_a and the error coefficients for the specific system types. These are shown in Table II-1.

System	K_p	K_v	K_a
Type 0	$\frac{1}{C_o} - 1$	O	O
Type 1	∞	$\frac{1}{C_1}$	O
Type 2	∞	∞	$\frac{1}{C_2}$

Table II-1. Effective Gain Constants for Typical System Types in Terms of Error Coefficients

The significance of the gain constant values presented in Table II-1 can be best illustrated by examining one of the system types.

For a type 1 system, the general transfer function may be written as

$$KG(s) = \frac{C}{E}(s) = \frac{K_v(\tau_1 s+1)(\tau_2 s+1)\ \dots\ (\tau_z s+1)}{s(\tau_a s+1)(\tau_b s+1)\ \dots\ (\tau_p s+1)} \tag{II-27}$$

The series integration represented by 1/s in Equation (II-27) implies that C(t) behaves in the steady-state condition as the integral of E(t). That is, if E(t) is a constant signal, then C(t) displays a constant velocity characteristic. With unity feedback around KG(s) the overall system will have no positional error; hence, $K_p = \infty$ in Table II-1. The system will display a velocity-lag error which is inversely proportional to the error coefficient, C_1. Therefore, high K_v means low steady-state velocity error in a type 1 system. If a step acceleration input is applied to the system, the error will not approach any finite value at the steady-state condition; hence, $K_a = 0$. Note that this does not mean that $C_2 = \infty$. K_a and C_2 possess the inverse relationship given in Table II-1 only in a type 2 system. C_2 for a type 1 system will have a finite value.

It is the hope of the author that the foregoing discussion has emphasized the usefulness of the root locus method as a tool for feedback control system analysis. With the salient features of this method in hand, a discussion of series dipole compensators and their effect on the root locus is in order.

CHAPTER III

SERIES DIPOLE COMPENSATION

Lead and Lag Dipole Compensators

Series compensation in the form of a phase lead or phase lag network is effective in altering the performance of a feedback control system. An uncompensated system may exhibit undesirable transient response characteristics or excessive steady-state error. These deficiencies in performance may be noted from an examination of the root locus plot of the system. It is possible to reshape the root locus or relocate the closed-loop roots by the addition of dipole compensating networks.

The intelligent selection of a lead or lag compensator for a particular system is enhanced by a knowledge of the general effects of these compensators on the system. This chapter will deal with these effects.

The transfer function of a dipole compensator is given by

$$G(s) = \frac{\tau_2(s + \frac{1}{\tau_2})}{\tau_1(s + \frac{1}{\tau_1})} \qquad \text{(III-1)}$$

where the relative magnitudes of the time constants, τ_1 and τ_2, dictate whether a circuit described by Equation (III-1) is a lead or a lag network.

If $\tau_2 > \tau_1$, then a lead network results. If $j\omega$ is substituted for (s) in Equation (III-1) and the phase angle and gain of G(s) are evaluated as a function of (ω), the plot of Figure III-1 is obtained.

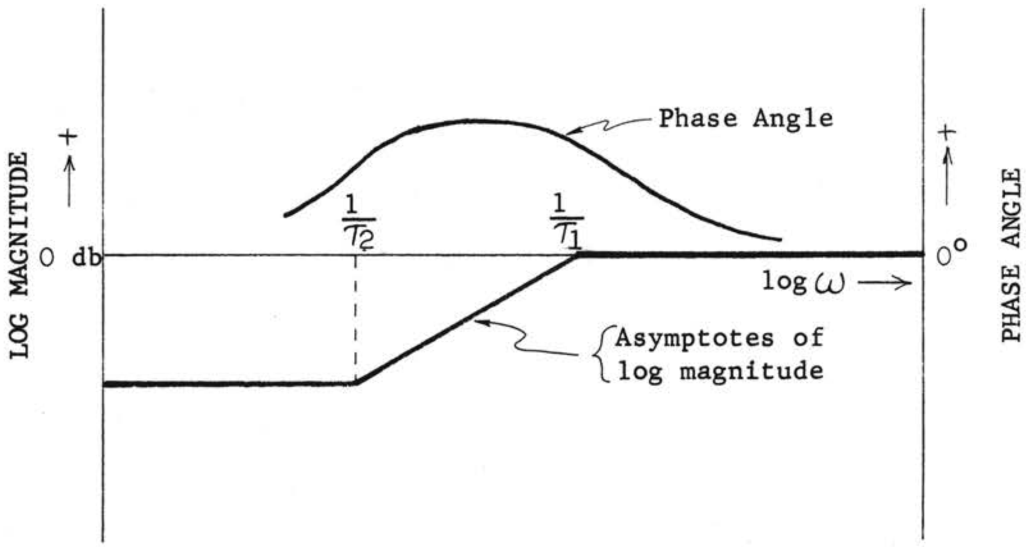


Figure III-1. Logarithmic Gain-Phase Plot of a Lead Dipole

$$G(j\omega) = \frac{\tau_2(j\omega + \frac{1}{\tau_2})}{\tau_1(j\omega + \frac{1}{\tau_1})} \qquad \text{where } (\tau_2 > \tau_1)$$

It can be seen that a lead dipole acts as a high-pass filter and, in general, intensifies the effects of high frequencies in a control system while attenuating the lower frequencies. Since stability is normally critical at higher frequencies, the lead dipole is used frequently to improve stability and transient performance.

A corresponding representation of a lead dipole on the s-plane is given in Figure III-2.

If $\tau_1 > \tau_2$, Equation (III-1) represents a lag dipole compensator and the log plot of such a device is shown in Figure III-3.

Here it is seen that a lag dipole acts as a low-pass filter attenuating the higher frequencies and thus has a significant effect in

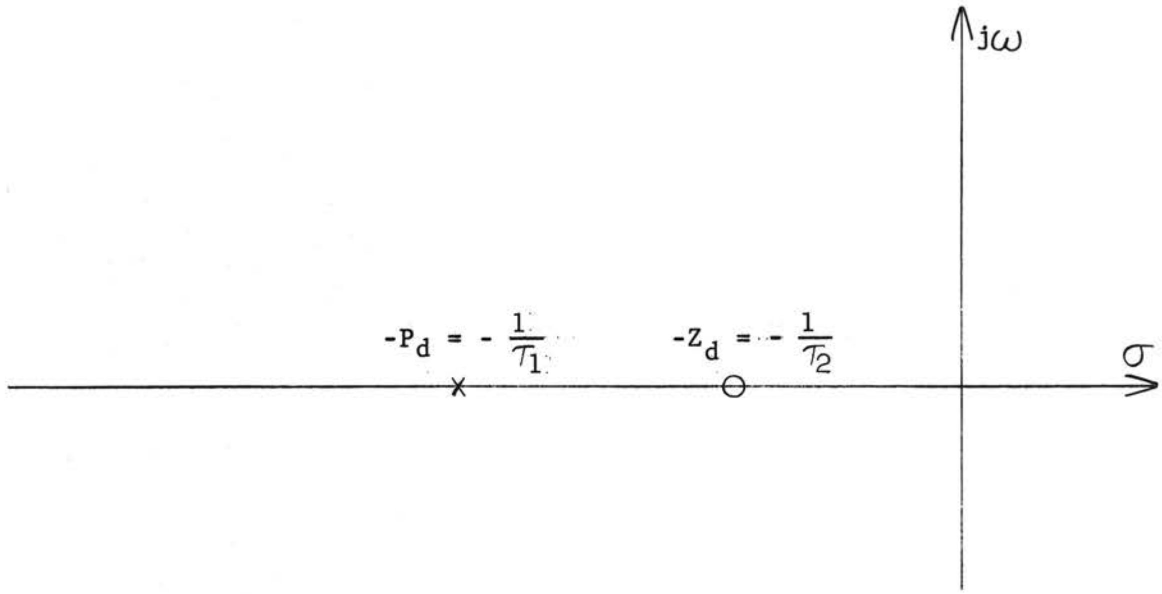


Figure III-2. Pole and Zero Representation of a Lead Dipole,

$$G(s) = \frac{\tau_2(s + \frac{1}{\tau_2})}{\tau_1(s + \frac{1}{\tau_1})} = \frac{K_d(s + Z_d)}{(s + P_d)} \quad \text{where } K_d = \frac{\tau_2}{\tau_1}$$

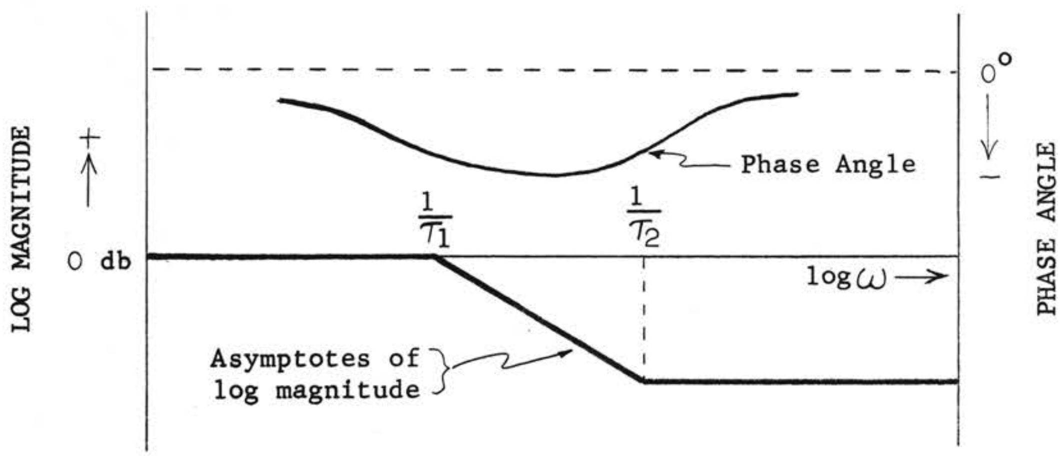


Figure III-3. Logarithmic Gain-Phase Plot of a Lag Dipole,

$$G(j\omega) = \frac{\tau_2(j\omega + \frac{1}{\tau_2})}{\tau_1(j\omega + \frac{1}{\tau_1})} \text{ where } (\tau_1 > \tau_2).$$

altering the low frequency operation of the system. Hence, steady-state performance can frequently be improved through the use of a lag dipole.

The s-plane plot of a lag dipole is shown in Figure III-4. Note that the only difference between the lag dipole and the lead dipole of Figure III-2 is a contraposition of the pole and zero.

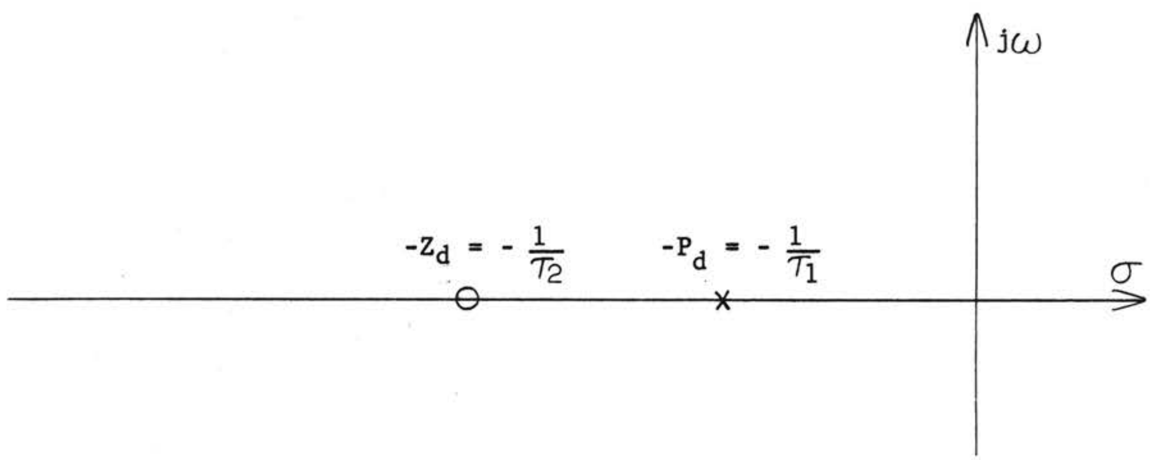


Figure III-4. Pole and Zero Representation of a Lag Dipole,

$$G(s) = \frac{\tau_2(s + \frac{1}{\tau_2})}{\tau_1(s + \frac{1}{\tau_1})} = \frac{K_d(s + Z_d)}{(s + P_d)} \quad \text{where } K_d = \frac{\tau_2}{\tau_1}$$

General Effects of Dipole on the Root Locus
and System Performance

It can be expected that the addition of a dipole compensator in series with the main control element will modify the characteristics of the uncompensated system. This alteration of the system performance will be evidenced by the movement of the closed-loop roots on the root locus plot of the system. Conversely, the addition of the pole and zero of a dipole compensator to an existing root locus will affect the location of the closed-loop roots, thereby influencing the system characteristics.

The relocation of the closed-loop roots can be accomplished in either of two ways or both. First, the roots may be shifted along the root locus branches while the branches themselves remain relatively unchanged. Second, the root locus itself may be translated on the s-plane, thereby forcing the roots to be relocated. In addition, the gain constant may, in some cases, be increased without significant movement of the roots. In this manner, steady-state error is reduced while the transient response of the system remains unaltered.

Any one of the methods just mentioned for relocating the closed-loop roots can be carried out by the proper placement of a dipole compensator on the root locus of the uncompensated system.

The specific effects of a dipole on a given system are, in general, unique for that system. It is difficult, therefore, to develop detailed rules which describe these effects for systems at large. However, certain fundamental properties of the root locus allow the listing of several helpful items.

The following changes will always occur in the root locus plot of a system with the addition of a dipole:

1. One additional open-loop pole—zero pair will be established on the negative real axis.
2. One additional root locus branch will be established.
3. One additional closed-loop root will be established. This root may appear as a single real root or combine with an

existing root of the system to form a complex conjugate pair.

Other changes in the root locus may occur depending on the specific system and the location of the dipole on the s-plane. Note that the difference between the number of poles and zeros does not change, thus leaving the number of root locus branches moving toward zeros at infinity and the direction of the asymptotes of these branches unchanged.

Although the intent of the present discussion is to treat the general effects of dipole placement on the root locus, the study of some specific example is almost mandatory in order to illustrate these effects. Naturally, some generality is lost, but a detailed discussion of a relatively simple system should prove worthwhile.

Assume that a series dipole compensator is to be applied to a type 0 system with an open-loop transfer function of the form

$$KG(s) = \frac{K}{(s + P_1)(s + P_2)(s + P_3)} \tag{III-2}$$

where the open-loop poles P_2 and P_3 are complex.

The uncompensated system will possess one real and two complex closed-loop roots as shown by the symbol $\otimes$ in Figure III-5(a). The roots shown assume a value of gain low enough to assure stability of the system. An increased gain would cause the complex roots to move into the right-half s-plane and thus cause the system to become unstable.

The addition of a lag dipole network of the form

$$G_d = \frac{K_d(s + Z_d)}{(s + P_d)} \tag{III-3}$$

causes a shift in the root locus of Figure III-5(a) and a relocation of

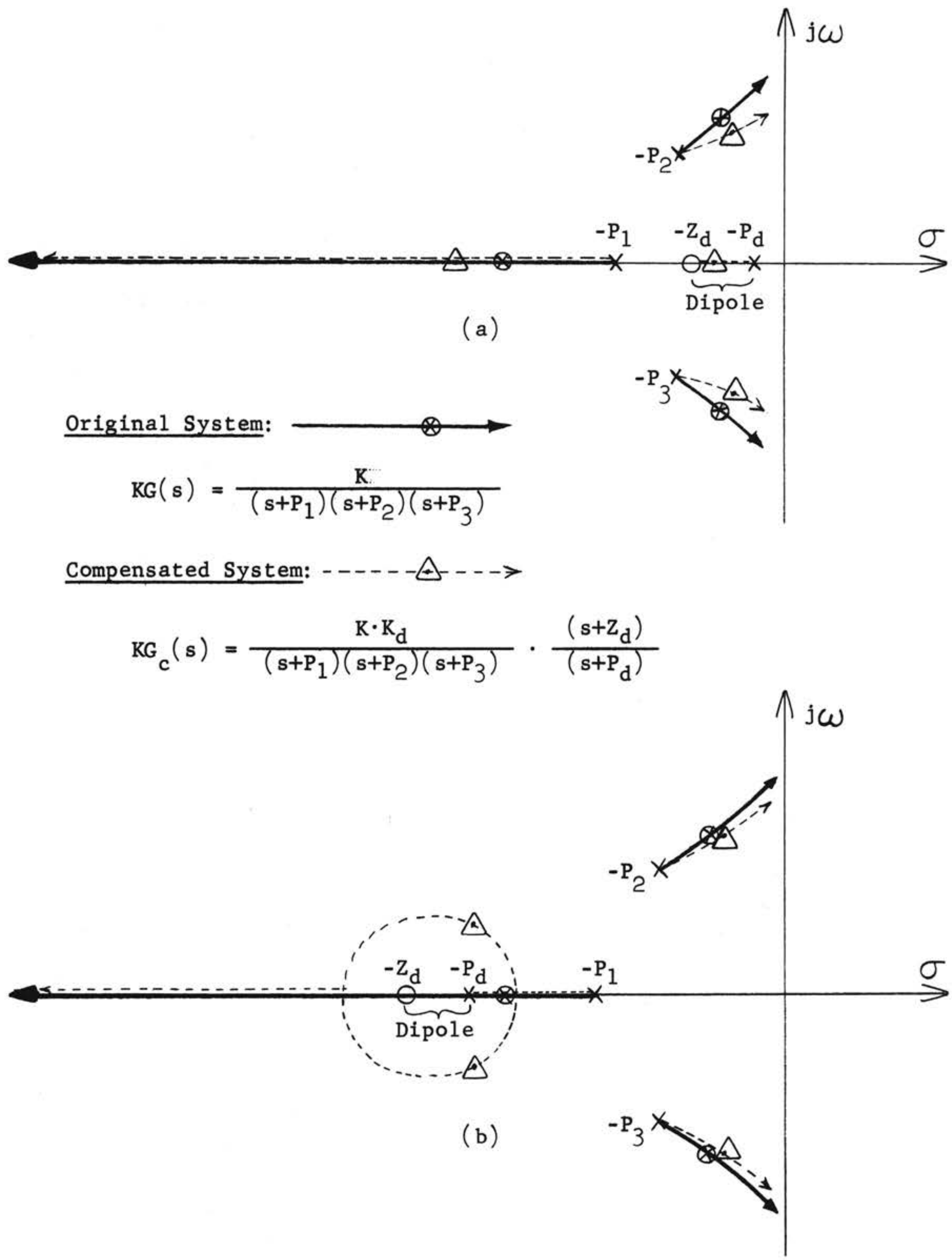


Figure III-5. Root Locus Plots Illustrating General Effects of Lag Dipole Placement the closed-loop roots. Assuming the composite gain constant $K \cdot K_d$ takes the original value of K, the new closed-loop roots are indicated by the symbol $\triangle$. One new real root has been added and it resides between the pole and zero of the dipole pair.

A comment is required at this point to clarify the method in which the gain constant $K \cdot K_d$ is held equal to the original value of K alone. From a comparison of Equations (III-1) and (III-3) the dipole compensator gain K_d is a function of the pole and zero values as

$$K_d = \frac{\tau_2}{\tau_1} = \frac{P_d}{Z_d} \tag{III-4}$$

Then, in the present example of a lag dipole, $K_d < 1$. For $K \cdot K_d = K$ (original), the gain constant K of the original system must be increased by a factor Z_d/P_d. This reduces steady-state error without sacrificing stability considerations.

If the original gain K were held constant, the complex roots of the compensated system would move back along the branches toward the complex open-loop poles, $-P_2$ and $-P_3$. The relative stability of the system would be improved due to an increased damping factor.

In this simple example, the movement of the roots due to the insertion of the dipole can be predicted by an examination of the dipole's angle contribution. An enlarged representation of the situation appears in Figure III-6.

Before the dipole is added, the root R_o occupies a position where the sum of the angles from the open-loop poles equals 180 degrees. The addition of the dipole furnishes a net angle $\phi = \theta_p - \theta_z$ which requires that a new position at R_c be assumed by the root. This new position

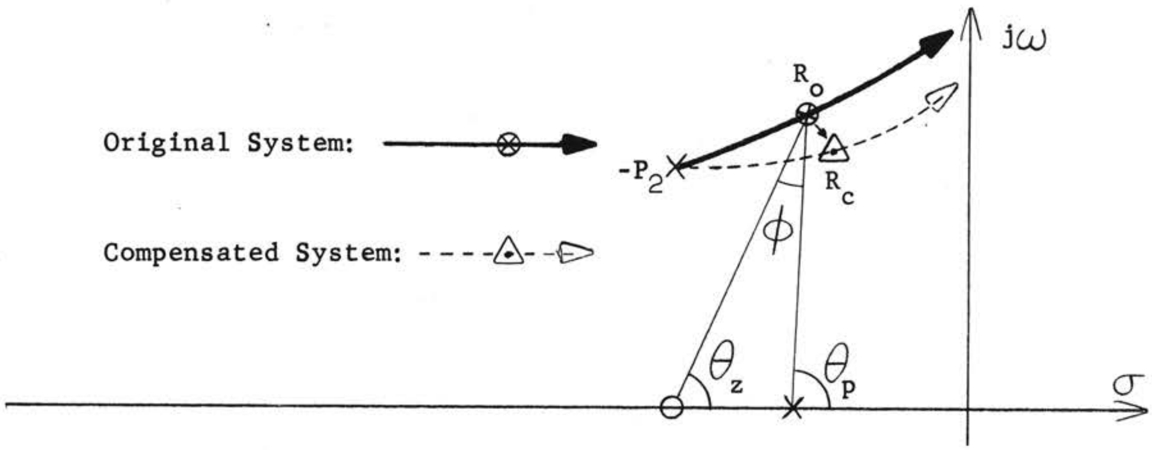


Figure III-6. Angle Contribution of a Dipole as it Effects Movement of a Closed-Loop Root.

compensates for the angle ϕ by moving in a general clockwise fashion about $-P_2$ until the position experiences a net angle of 180^o again. It can be seen that if $-P_2$ had been replaced with a zero, the root would have shifted in a counterclockwise fashion. No general statement concerning the movement of a root due to the addition of a dipole can be derived from this example, since each root locus configuration will force the root to move in a particular fashion.

Placing the lag dipole farther away from the origin results in a very different root locus plot as shown in Figure III-5(b). With gain $K \cdot K_d$ fixed at the value of the first example, two more complex roots have been produced. This conjugate pair, however, will not have a predominant effect upon the transient response of the system, since the decay rate is approximately four times that of the complex roots nearer the imaginary axis.

Notice that the shift in the root locus branches at the open-loop poles $-P_1$ and $-P_2$ is reduced. This is due to the fact that the dipole contributes less net angle to these branches as it is placed farther away from the origin. At great distances from the origin the dipole will have a negligible effect on the root locus, and thus no compensating effect on the system performance.

Steady-state error will still be reduced somewhat but not to the extent as when the dipole was near the origin in Figure III-5(a). This is due to a decrease in the ratio Z_d/P_d as the dipole departs from the origin.

The effect of inserting a lead dipole into the system is shown in Figure III-7. Again, two cases have been investigated. The lead dipole has been placed close to the origin in Figure III-7(a), whereas Figure III-7(b) illustrates the case where the dipole is relocated some distance from the origin.

A more severe change in the locus can be observed in the first case due to the larger net angle contribution of the dipole to the complex branches and their roots. The zero contributes the larger angle now and the root associated with the pole, $-P_2$ must rotate in a counterclockwise fashion about the pole to resatisfy the angular condition of the locus.

A slowly decaying closed-loop root is produced within the dipole. This root can be damaging to the transient response if the exponential term in the transient expression has a large coefficient. For a step input, this coefficient is the residue of C(s) in the root where, in the present example, C(s) is given by

$$C(s) = \frac{1}{s} \cdot \frac{K \cdot K_d(s + Z_d)}{(s + R_1)(s + R_2)(s + R_3)(s + R_4)} \qquad \text{(III-5)}$$

Keeping the root close to the zero of the dipole reduces the factor $(s + Z_d)$ thus keeping the residue small, and the coefficient of the exponential term becomes negligible. A high gain constant value is required to accomplish this.

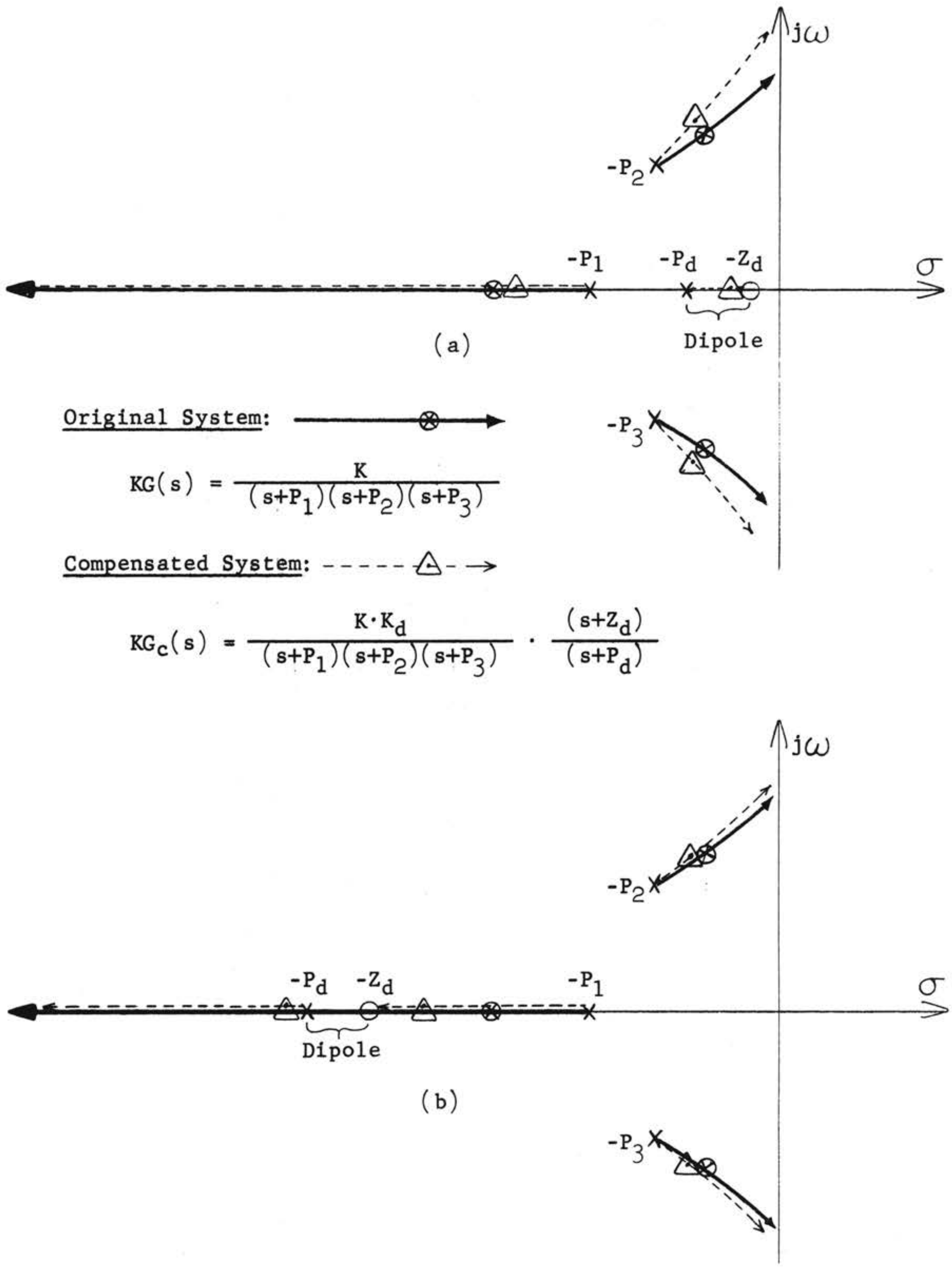


Figure III-7. Root Locus Plots Illustrating General Effects of Lead Dipole Placement

The steady-state error of the system is increased due to the unfavorable ratio Z_d/P_d which forces the effective gain constant K_p, of the system to be reduced. The gain could be increased to improve steady-state operation but instability might result.

Figure III-7(b) indicates that when the lead dipole is positioned farther away from the origin, the effect on the locus shape is reduced. The real roots have also been relocated away from the origin thus yielding fast decay for these roots in the transient response.

Small change is seen in the complex branches and as the dipole is shifted farther out from the origin, the dipole will have less effect on the system.

The investigation of the foregoing examples has assumed a fixed separation between the pole and zero of the dipole. Interesting effects on the root locus are observed when either the pole or zero is held fixed and the other element varied. This procedure and others involving parameter variation will be taken up in connection with some typical systems in Chapter IV.

CHAPTER IV

DIPOLE VARIATION TECHNIQUES

The Root Locus Family

It is of fundamental importance that any design method involving the root locus yield a maximum amount of necessary information about the system performance with the least time and effort. The term "necessary information" would have to be further defined in the light of a specific compensator design problem. It might be required that only the closed-loop root locations of a specific compensated system for certain dipole positions be known. This would imply compensation with no gain adjustment in the system and the root locus plot is rather easily constructed. If, however, "necessary information" constitutes a knowledge of the closed-loop root locations with both dipole position and gain as variables, then the root locus plot is more involved and more time and effort is entailed in its construction.

In the latter of the two cases, both dipole position and system gain are available to the designer for the improvement of system performance and a family of root locus plots is required. This establishes the location of all closed-loop roots produced by the many combinations of gain and dipole position. The procedure is to select a lead or lag dipole, this choice being determined by the specific performance improvement desired. The root locus is then constructed as a function of system gain for a specific dipole position.

This process is repeated for successive dipole positions, thus yielding a family of root locus plots. Normally, after two or three dipole positions have been selected, and the locus for each has been constructed, certain trends in the shifting of the locus can be detected and a more judicious selection of additional dipole positions can be effected.

To illustrate the root locus family technique, two examples have been investigated. The uncompensated systems involved were not selected on the basis of undesirable performance properties, but rather, on the strength of their root locus configurations and the effects obtained when dipole variation was imposed.

Each root locus plot reproduced in this paper was constructed with the spirule at twice the scale of that shown. Attendant graphical inaccuracies can be expected; particularly in those regions close to a pole or zero.

A type 1 system with an open-loop transfer function

$$KG(s) = \frac{K(s + 4)}{s(s + 2)(s + 2 + j)(s + 2 - j)} \tag{IV-1}$$

becomes the first example of the root locus family technique.

Figure IV-1 shows the root locus plot of the uncompensated system. Also shown are the lead and lag dipole positions which will be imposed on the system. Notice that only one-half of the total root locus configuration is indicated in Figure IV-1. Since the locus possesses symmetry with respect to the real axis, no pertinent information is lost by considering only those branches in the second quadrant and on the real axis.

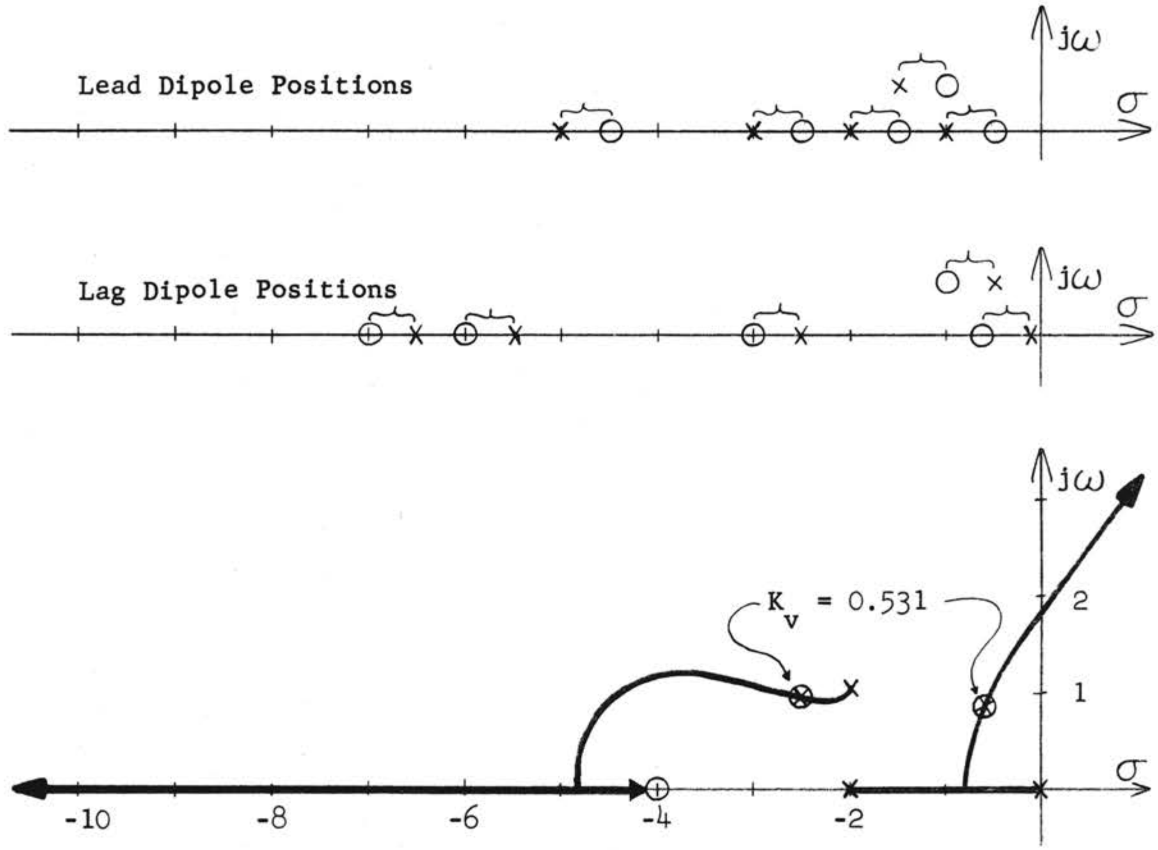


Figure IV-1. Root Locus of Uncompensated System with

$$KG(s) = \frac{K(s + 4)}{s(s + 2)(s + 2 + j)(s + 2 - j)}$$

This plotting convention will be extended through the remainder of this paper.

The uncompensated system with the open-loop function given by Equation (IV-1) possesses one open-loop zero and four open-loop poles. There will be four closed-loop roots with the possibility of all these being complex for certain values of system gain. One such gain value is K = 1.33, and the location of the closed-loop roots for this gain are indicated in Figure IV-1. The effective velocity gain constant, K_v, is fixed for each of the various dipole positions considered. That is, with K_v held constant, the error coefficient $C_1 = 1/K_v$ will remain constant and the velocity-lag error will be fixed regardless of the dipole position. K_v may be found in the present example as

$$K_v = \frac{4K}{(2)(5)} = 0.531 \tag{IV-2}$$

After series dipole compensation is accomplished, the open-loop transfer function of the compensated system becomes

$$KG_c(s) = \frac{K(s + 4)(s + Z_d)}{s(s +2)(s + 2 + j)(s + 2 - j)(s + P_d)} \tag{IV-3}$$

The velocity gain constant K_v may now be found from

$$K_v = \frac{4\ Z_d}{(2)(5)P_d}\ K = 0.4\ \frac{Z_d}{P_d}\ K \tag{IV-4}$$

Since the ratio Z_d/P_d will, in general, vary with successive dipole positions, the gain K must vary inversely to the ratio Z_d/P_d to maintain $K_v = 0.531$.

Figure IV-2 is the root locus family produced when the lead dipole compensator is varied through five distinct positions. The uncompensated system is also indicated for reference. The information concerning the root locus variation is presented in two areas of Figure IV-2. At the upper portion of the figure are representations of the negative real axis for the uncompensated system and for the five lead dipole positions. These axes possess the same scale as the lower axis in the figure where the complex loci are shown. On each real axis, the locus breakaway and return points are indicated, and the position of the lead dipole is shown in relation to the existing real poles and zeros of the uncompensated system. In addition, the closed-loop roots for $K_v = 0.531$ are shown for each dipole position.

The complex portions of the locus for each lead dipole position is

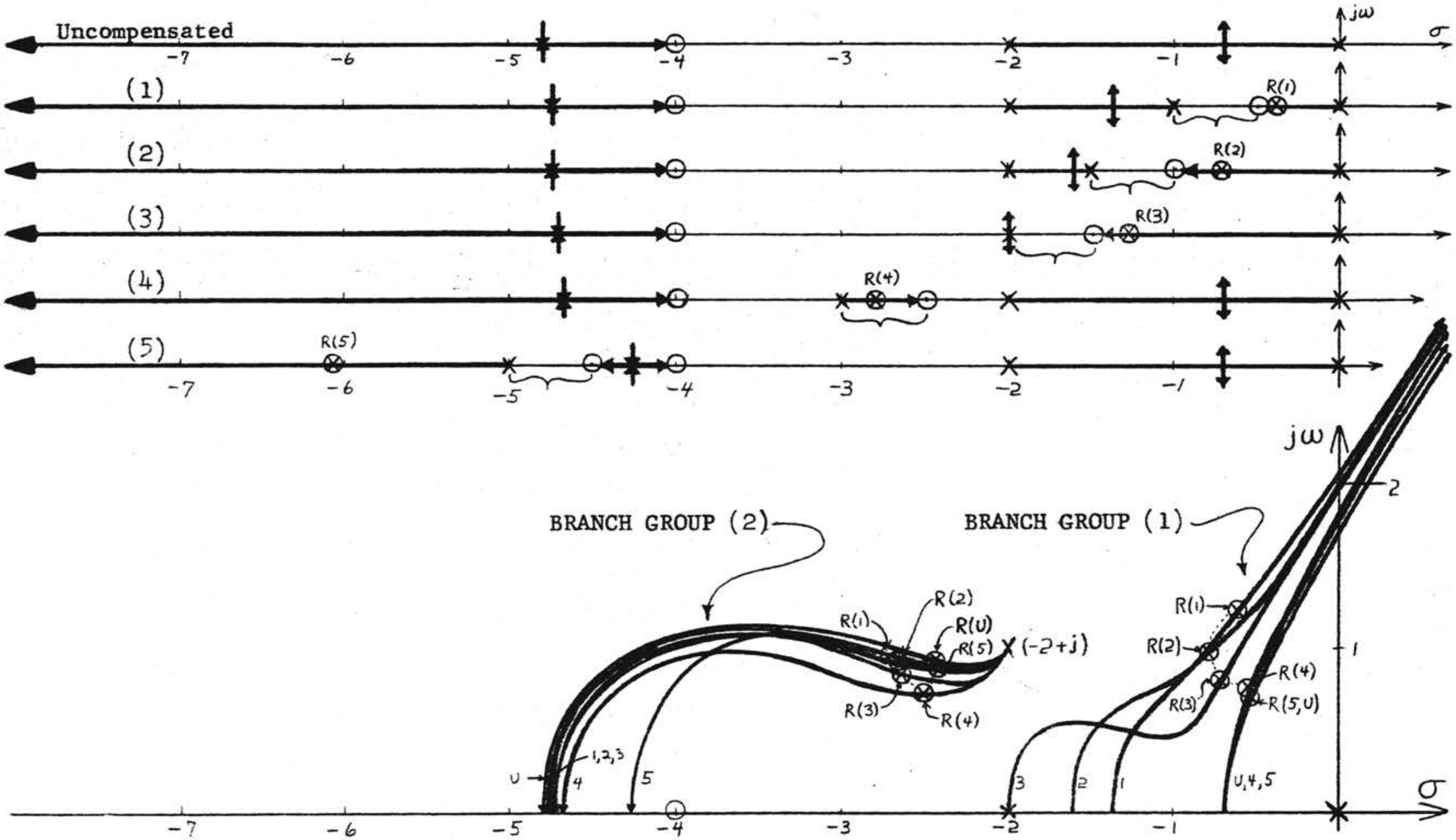


Figure IV-2. Root Locus Family for $KG_c(s) = \dfrac{K(s+4)}{s(s+2)(s+2+j)(s+2-j)} \cdot \dfrac{(s+Z_d)}{(s+P_d)}$ with Variation of a Lead Dipole Having Fixed Pole-Zero Separation shown in the lower half of Figure IV-2. The positions of the complex closed-loop roots for each compensation at $K_v = 0.531$ are indicated. Actually, only one member of each complex root pair is shown.

A discussion of the significant effects of the dipole variation on the root locus will be divided on the basis of (1) locus alterations on the negative real axis and (2) effects on the complex locus branches.

Certain observations may be made concerning the locus changes on the negative real axis. One additional closed-loop root is produced by the lead dipole. This root shifts away from the origin as the dipole position progresses further out the negative real axis. The lead dipole affects the locus breakaway and return points only when the dipole is positioned in close proximity to these points.

An examination of the loci in the complex region will verify the statement in Chapter III that the addition of a dipole does not change the direction of the asymptotes for the locus branches moving to zeros at infinity. These directions in the present example are $\pm 60^\circ$ and -180°.

Another interesting point is that the intersection of the asymptotes with the negative real axis does not change with the variation of a dipole with fixed pole-zero separation. Rule No. 8 from Chapter II, for the construction of the root locus, is repeated here for convenience.

The intersection of the asymptotes with the real axis is given by

$$\frac{\sum \text{real parts of poles} - \sum \text{real parts of zeros}}{p - z} \qquad \text{(IV-5)}$$

Obviously, the factor (p - z) is unaffected by the introduction of a dipole. In the present example, (p - z) = 3. The numerator, however, is a function of the dipole pole-zero separation. In the case under consideration, the introduction of a lead network would cause a decrease in the value of the numerator of Equation (IV-5) and the asymptote intersection would shift to a more negative value from that of the uncompensated system. The addition of a lag network would have the opposite effect. That is, the intersection would shift to the right.

In the present example, the dipole separation is 0.5 thus causing the asymptote intersection to shift to the left by an amount $0.5/3 = 0.166$. This condition is evidenced in Figure IV-2 by the locus branches of the compensated system crossing the imaginary axis at somewhat higher values of $j\omega$ than does the uncompensated locus branch.

Another significant point is the larger effect on branch group (1) produced by dipole positions (1), (2) and (3), compared to the almost negligible effect of these same dipole positions on branch group (2). Conversely, it can be seen that dipole position (5) affects branch group (1) only slightly. As was brought out in Chapter III, these results are due to the relatively small angle contribution of a dipole to the locus in a region remote from the dipole position.

Due to the large time constant effects, those changes in branch group (1) will have the predominant role in altering the transient performance. Considering the movement of the closed-loop roots versus dipole position, dipole position (3) produces the greatest damping of the system. Also, a slightly faster decay would decrease the settling time for a step input. If fast response to a step input were desired and the maximum overshoot of the output was not critical, then dipole position (1) would be a logical choice. It should be observed that dipole position (1) produces a negative real root close to the origin.

The decay of the transient term produced by this root will be relatively slow, thus tending to reduce the speed of response. However, the magnitude of its coefficient in the transient expression, $\frac{C}{R}(s)$, will be small, due to its proximity to the zero of the dipole, and thus its effect on the total transient response will not be as great as might be expected.

Note that gain adjustment could also be employed to relocate the closed-loop roots anywhere along the loci in order to satisfy system requirements. In each case, a decrease in gain would improve system stability but the speed of response would be diminished and steady-state error would be increased.

A series lag compensator can be expected to produce somewhat different results in terms of reshaping the root locus and relocating the closed-loop roots. If various lag dipole positions are used to produce a second root locus family, the plot would appear as Figure IV-3. Dipoles with fixed pole-zero separation have been employed in this figure and their positions relative to the uncompensated system are shown in Figure IV-1.

In a fashion similar to the lead dipole variation in Figure IV-2, representations of the locus on the negative real axis have been separated from the locus branches in the complex region. For each lag dipole position the closed-loop roots for $K_V = 0.531$ have again been indicated. This fixes the velocity-lag error of the system and only the transient performance will be affected by successive dipole positions.

As with the introduction of a lead dipole, series compensation with a lag dipole produces an additional closed-loop root. For this particular example, the root will always reside on the negative real axis for all dipole positions and all values of gain. The movement of this real

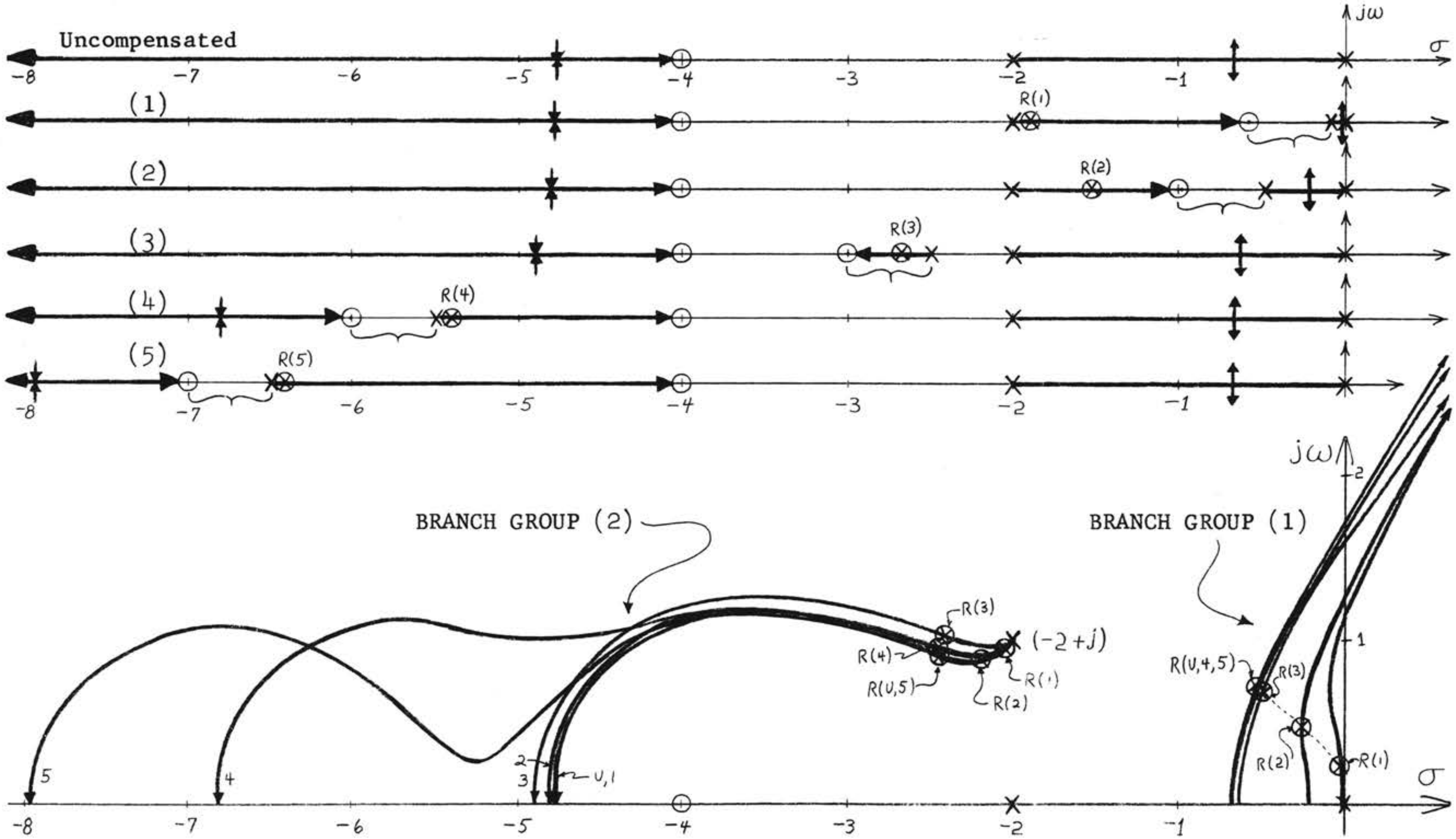


Figure IV-3. Root Locus Family for $KG_c(s) = \dfrac{K(s + 4)}{s(s + 2)(s + 2 + j)(s + 2 - j)} \quad \dfrac{(s + Z_d)}{(s + P_d)}$ with

Variation of a Lag Dipole Having Fixed Pole-Zero Separation root is, in general, dictated by the position of the lag dipole and as the dipole progresses out the negative real axis, the closed-loop root also positions itself farther away from the origin. The locus breakaway and return points are affected only when the lag dipole is positioned near one of these points.

Turning now to the complex branches of the locus, it can be seen from Figure IV-3 that the asymptotes of the locus branches moving to zeros at infinity have not changed their angular orientation from that of the uncompensated system. However, the real axis intersection of these asymptotes has shifted to the right by a value of 0.5/3 due to the 0.5 separation of the pole and zero in the lag dipole. This causes the loci of branch group (1) to cross the imaginary axis at lower values of $j\omega$ than does the uncompensated locus branch.

Dipole positions (1) and (2) exercise the greatest influence on branch group (1), while branch group (2) is dominated by dipole positions (3), (4) and (5). Actually, no desirable compensation has been achieved through the use of the lag dipole in this particular case. It is evident from the relocation of the dominant closed-loop roots of branch group (1) that the transient performance of the system has not been improved. The roots R(1) and R(2) have been shifted closer to the imaginary axis, thereby increasing their time constant effect, and at the same time no increased system damping is observed. In fact, root R(1) would undermine the response of the system to a step input by creating a slowly decaying oscillatory component of low frequency.

It is worthwhile to note the effect of dipole position (5) on the system. Little or no influence on branch group (1) and the associated closed-loop root is evident. Also, in branch group (2) it can be seen that the locus for dipole position (5) is tending to realign itself with the uncompensated locus. If dipole positions were selected farther out on the negative real axis, the depression already evident in the branch for dipole position (5) would become more pronounced and the locus would closely parallel the locus of the uncompensated system. This statement ignores the small loop executed by the locus at the lag dipole which now would be positioned far to the left.

There is little possibility that any of the locus in this region would contain a closed-loop root, since the gain required to produce such a root would be very high and the roots of branch group (1) would be forced into the right-half s-plane, thus creating unstable system operation.

The foregoing examples have employed dipoles which possess fixed pole-zero separation. For the most part, the alterations of the root locus shape were not pronounced due to this rather small separation. Also, large translations of the closed-loop roots for a constant gain value were not observed.

More extensive results are obtained when the pole-zero separation is allowed to vary rather than varying both the pole and zero as a fixed device. This mode of dipole variation was applied to an uncompensated type 1 system with the open-loop transfer function

$$KG(s) = \frac{K}{s(s + 2 + j2)(s + 2 - j2)} \qquad \text{(IV-6)}$$

The root locus of the uncompensated system is shown in Figure IV-4. For all values of gain, the system will possess one real root and one pair of complex conjugate roots. These roots are indicated in Figure IV-4 for a velocity gain constant of $K_v = 1.48$.

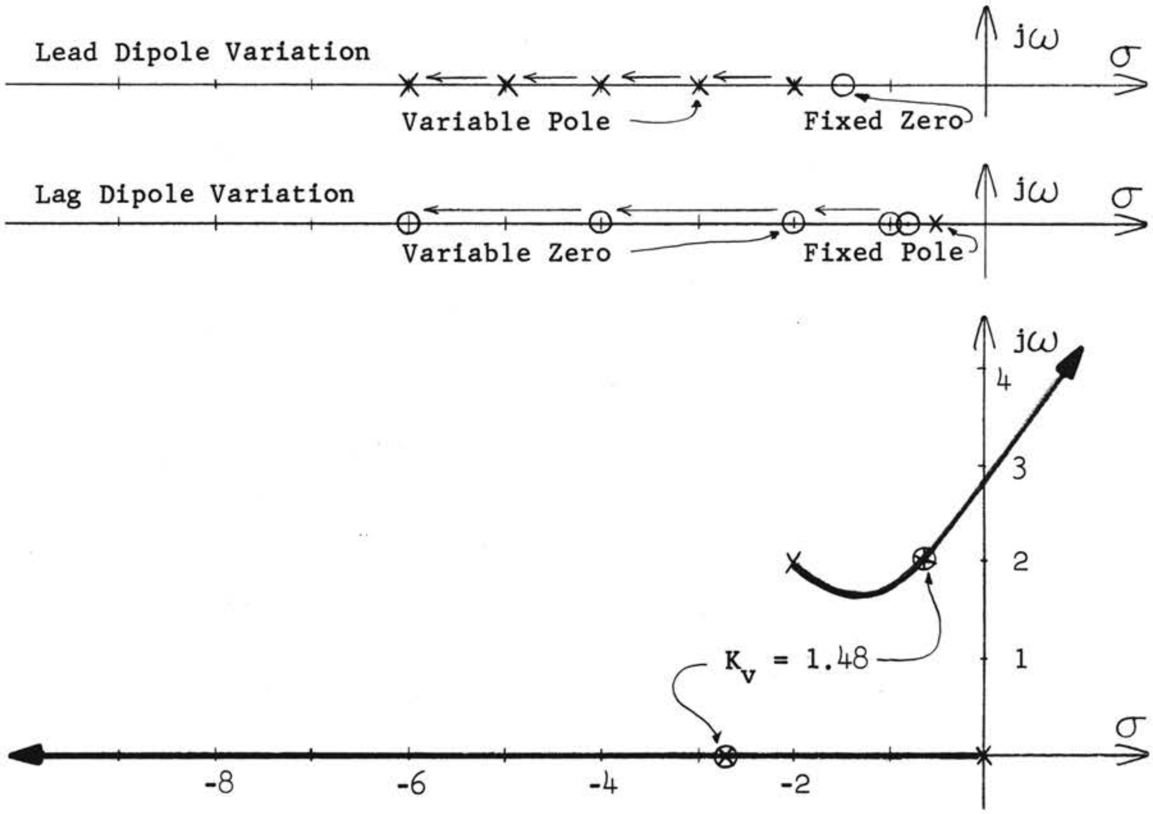


Figure IV-4. Root Locus of Uncompensated System with

$$KG(s) = \frac{K}{s(s + 2 + j2)(s + 2 - j2)}$$

It should be stressed that the scale selected for plotting the root locus for this, and the previous example, does not allow for very high values of gain. In a physical system, the time constants would be shorter than those proposed here and correspondingly higher values of gain would be obtained. Since the purpose of these plots is to indicate relative movement of the locus configurations, the scale selected plays no direct role in the investigation.

In the upper portion of Figure IV-4 are representations of the negative real axis with the various lead and lag dipole positions shown. Note that in each compensator either the pole or zero remains fixed while the other element of the dipole is varied. For instance, in the lead dipole the zero is fixed and the pole is varied through five distinct positions. In an opposite fashion, the lag dipole possesses a fixed pole and a variable zero.

To determine an appropriate position for the fixed zero or pole in the lead and lag dipoles, consideration must be given to the gain of the system. After series dipole compensation has been applied to the system of Equation (IV-6), the open-loop transfer function becomes

$$KG_c(s) = \frac{K(s + Z_d)}{s(s + 2 + j2)(s + 2 - j2)(s + P_d)} \qquad \text{(IV-7)}$$

The velocity gain constant is

$$K_v = \frac{Z_d}{8\ P_d} \cdot K \qquad \text{(IV-8)}$$

To allow a reasonable value of K_v and therefore acceptable velocity lag error, the ratio Z_d/P_d in Equation (IV-8) must not approach zero. To this end, the fixed zero in the lead dipole of Figure IV-4 must not be close to the origin. Another consideration is that a closed-loop root will be produced on the real axis somewhere between the fixed zero and the origin, since the uncompensated system contributes an open-loop pole at the origin. Placing the fixed zero of the lead dipole some distance from the origin will allow the closed-loop root to contribute a shorter time constant effect, thereby improving the transient response of the system.

A lag network with the pole placed near the origin will cause the ratio Z_d/P_d to be large, thus providing large velocity gain constant and low velocity lag error. In the extreme case, where the fixed pole of the lag dipole is located at the origin, a type 2 system results and no velocity lag error is produced. In the present example, the location of the fixed dipole elements have been determined with these points in mind.

Figure IV-5 is a root locus family obtained when lead dipole variation and gain variation are employed. As in the previous figures, the negative real axis is shown at the top of Figure IV-5 for each lead dipole position. Closed-loop roots are indicated for a velocity gain constant of $K_v = 1.48$. To provide this condition, the open-loop gain K must be varied as successive lead dipole positions are selected. An examination of Equation (IV-8) indicates the logic of this statement.

The addition of a lead dipole again adds one more closed-loop root to the three roots already possessed by the uncompensated system. For all values of gain, the additional root will be real, thus making two real roots for the compensated system. One of these roots remains in the region between the origin and the fixed zero of the dipole at $s = -1.5$. The time constant effect of this root will not be detrimental to the over-all transient response of the system as long as it lies close to the zero. The second real root progresses out the negative real axis, always appearing to the left of the variable pole of the lead dipole. Its relatively short time constant effect makes its contribution to the system transient response negligible compared with the other real root and the complex conjugate pair. Notice that the negative real axis has been contracted for dipole positions (4) and (5), so as to show the corresponding closed-loop roots at $s = -6.35$ and $s = -7.2$.

An examination of the complex portion of the root locus family reveals certain trends. The angle of departure of the locus branch from the open-loop pole at $s = -2 + j2$ increases as the variable pole of the lead dipole progresses out the negative real axis. This is to

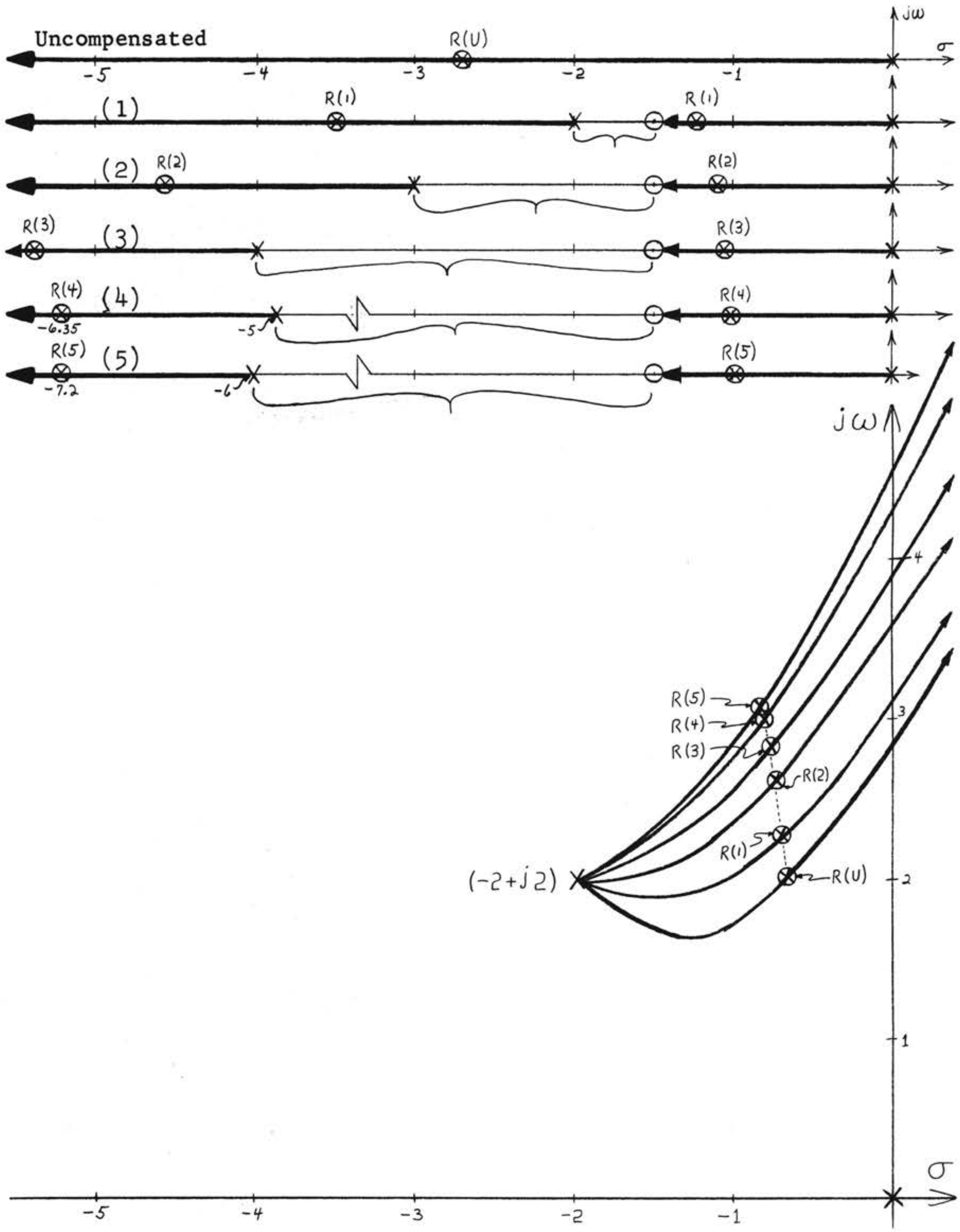


Figure IV-5. Root Locus Family for

$$KG_c(s) = \frac{K}{s(s+2+j2)(s+2-j2)} \cdot \frac{(s+Z_d)}{(s+P_d)}$$

with Variation of a Lead Dipole Having Variable Pole-Zero Separation be expected since the angle contribution of the dipole becomes predominantly the angle of the dipole zero alone, as the variable pole moves to the left. To counter this additional angle contributed by the zero, the locus in the vicinity of the complex pole must shift counterclockwise about the pole.

Another interesting observation concerns the asymptotes of the locus branches. The angle of these asymptotes has not changed, since the value (p - z) has not been altered. However, the intersection of the asymptotes with the real axis varies with the dipole position. This phenomenon did not occur in the previous example when the dipole possessed fixed separation. In the present case, however, the asymptote intersection shifts to the left exactly one-third of the shift executed by the variable pole. The factor one-third is derived from the quantity (p - z) = 3 and Equation (IV-5).

The lead dipole is not effective in improving the performance of the uncompensated system. The position of the closed-loop roots on the complex branches indicates that no appreciable change in the damping factor can be obtained without gain adjustment, and this procedure could have been applied easily to the uncompensated system. Also, the additional real root produced by the dipole introduces a longer time constant term than was provided by the single real root of the uncompensated system.

Phase lag compensation of this particular system produces more interesting results. Figure IV-4 indicated various lag dipoles with a fixed pole and a variable zero. When the root locus of the compensated system is constructed for each of these lag dipoles, the root locus family given by Figure IV-6 results. Closed-loop roots are again indicated for a velocity gain constant of 1.48. At this gain no real roots exist for the compensated system. Even the single real root which characterized the original system has been removed. For very low values of gain or for values of gain somewhat higher than $K_v = 1.48$, two real roots will exist but at least one pair of complex roots are produced at all values of gain. Since no real roots appear at $K_v = 1.48$, little discussion is required concerning the negative real axis representations at the top of Figure IV-6. Note that the breakaway and return points have been indicated.

Turning to the complex branches of Figure IV-6, certain interesting points may be shown. It may be seen that rather extensive reshaping of the root locus is accomplished with the lag dipole variation. Special attention should be called to the marked difference between the locus branches for dipole position (1) and dipole position (2). In the region about $s = -1 + j$ the locus branches for dipole position (1) appear above and below this point, whereas with a slight shift of the variable zero to dipole position (2), the locus branches assume vertical forms to the right and left of the region about $s = -1 + j$. Extensive plotting at larger scales indicates that when the variable zero of the lag dipole is located at about $s = -0.85$ the locus branches will become tangent near the point $s = -1 + j$. In addition, a velocity gain of approximately 1.42 will force the two closed-loop roots to reside at this tangency. The use of the spirule in this region did not allow a great deal of accuracy in plotting the locus, hence the approximate figures for gain and the tangency point.

The closed-loop roots at $K_v = 1.48$ have been indicated in Figure IV-6 along with the approximate paths of these roots when K_v is held

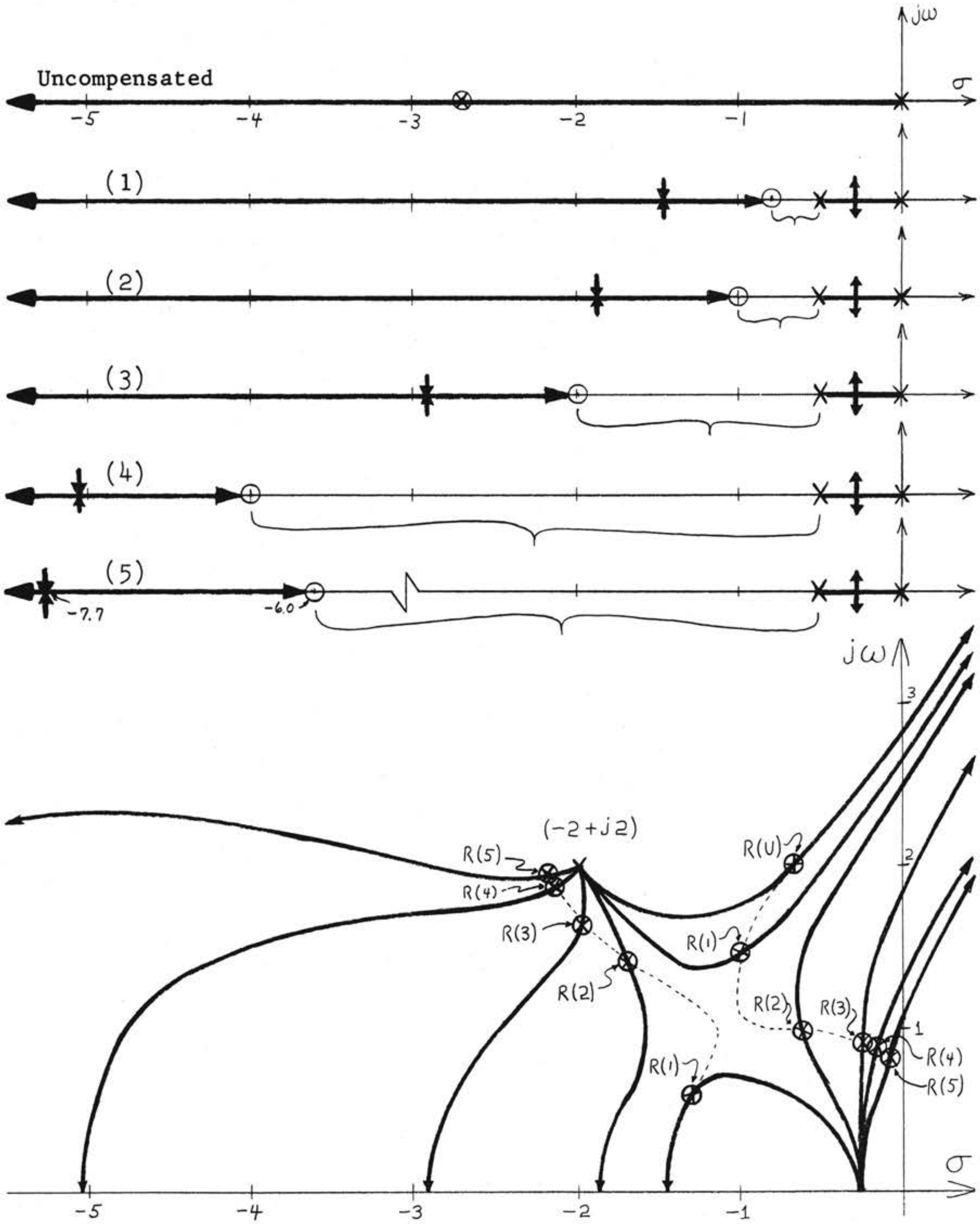


Figure IV-6. Root Locus Family for

$$KG_c(s) = \frac{K}{s(s + 2 + j2)(s + 2 - j2)} \cdot \frac{(s + Z_d)}{(s + P_d)}$$

with Variation of a Lag Dipole Having Variable Pole-Zero Separation constant and the dipole position is varied.

If system damping were to be increased, suitable compensation for the original system at $K_v = 1.48$ would be that lag dipole which would force both pairs of closed-loop roots to reside near $s = -1 + j$. This would provide more than adequate damping. For this compensation the lag dipole would be

$$G_d(s) = \frac{s + 0.85}{s + 0.5} \tag{IV-9}$$

The rotation of the locus departure angle from the complex open-loop pole $s = -2 + j2$ is due to the decreasing angle contribution of the variable zero as it shifts further from the origin. In this case, the locus near the pole at $s = -2 + j2$ must accept a decreasing angle contribution from this pole in order to satisfy the fundamental angular condition. Therefore, the locus branches display a clockwise rotation about $s = -2 + j2$ as the zero of the dipole moves to the left.

The asymptotes of the locus plot do not experience any change in direction, but the intersection with the real axis does shift to the right with each successive movement of the variable zero to the left.

The four examples just shown have illustrated the root locus family technique for design of series lead and lag dipoles. Large variations in the performance characteristics of the systems were not always obtained specifically in each case but further variation of gain or readjustment of dipole configuration was indicated for such variations. Also, certain dipole types were ruled out as possibilities for consideration.

Admittedly, the root locus family technique is laborious when locus plots must be constructed by hand. However, with computers being applied to this problem, the most prohibitive disadvantage of the technique is eliminated.

Retaining system gain as an active variable in the root locus family technique is of major importance. Although the translation of the closed-loop roots in the s-plane at a fixed gain for various dipole positions is valuable information in itself, the effect of gain adjustment for any dipole selection completes the picture.

When gain is to remain unchanged in a system, more rapid plotting techniques may be applied. These will be discussed in the next section.

Single Parameter Variation

The root locus plot as a function of system gain can actually be termed a special case of a generalized technique. (18,19). A plot of the root locus with a system parameter as variable can be carried out in much the same fashion as when system gain is allowed to vary. However, for parameter variation, a corrected expression must be formulated which is analogous to the familiar open-loop function, KG(s). Furthermore, it will be shown that two elements such as a pole and zero of a dipole can be used as variables if they are related by a constant.

To illustrate this procedure, and simultaneously show its application to dipole compensator design, the system described by Equation (IV-6) will be employed. In general form, the open-loop transfer function can be expressed as

$$KG(s) = \frac{K}{s(s + P_1)(s + P_2)} \qquad \text{(IV-10)}$$

The application of dipole compensation transforms Equation (IV-10) into

$$KG_c(s) = \frac{K}{s(s + P_1)(s + P_2)} \cdot \frac{s + Z_d}{s + P_d} \tag{IV-11}$$

The choice now arises as to which element of the dipole will be varied. This decision would be based on the performance requirements of the compensated system. For future comparison purposes with a previous example, suppose a lag network with fixed pole and variable zero is the dipole configuration selected.

Manipulation of Equation (IV-11) into the closed-loop transfer function yields

$$\frac{C}{R}(s) = \frac{KG_c(s)}{1 + KG_c(s)} \tag{IV-12}$$

$$= \frac{K(s + Z_d)}{s(s + P_1)(s + P_2)(s + P_d) + K(s + Z_d)} \tag{IV-13}$$

The denominator of Equation (IV-13), when set equal to zero, becomes the characteristic equation of the unity feedback system, thus

$$s(s + P_1)(s + P_2)(s + P_d) + K(s + Z_d) = 0 \tag{IV-14}$$

The open-loop gain, K, can be written in terms of the velocity gain constant, K_v, and the open-loop poles and zeros as

$$K = \frac{K_v P_1 P_2 P_d}{Z_d} \tag{IV-15}$$

Combining Equations (IV-14) and (IV-15), the characteristic equation becomes

$$s(s + P_1)(s + P_2)(s + P_d) + \frac{K_v P_1 P_2 P_d}{Z_d}(s + Z_d) = 0 \tag{IV-16}$$

Recalling that the root locus is a plot of the values of (s) which satisfy Equation (IV-16) as K_v is varied, we must now consider K_v as a constant and Z_d as the variable quantity.

Manipulation of Equation (IV-16) yields

$$K_v(P_1P_2P_ds)\left[1 + \frac{Z_d\left[s(s + P_1)(s + P_2)(s + P_d) + K_vP_1P_2P_d\right]}{K_vP_1P_2P_ds}\right] = 0 \qquad \text{(IV-17)}$$

Those values of (s) satisfying Equation (IV-17) when Z_d is a variable must also satisfy the equation derived by equating to zero the bracketed factor of Equation (IV-17).

$$1 + \frac{Z_d\left[s(s + P_1)(s + P_2)(s + P_d) + K_vP_1P_2P_d\right]}{K_vP_1P_2P_ds} = 0 \qquad \text{(IV-18)}$$

Comparison of the left member of Equation (IV-18) with the denominator of the right member of Equation (IV-12) will indicate that a new open-loop function has been derived which will replace $KG_c(s)$ for purposes of plotting the root locus. This derived open-loop expression is

$$KG'(s) = \frac{Z_d\left[s(s + P_1)(s + P_2)(s + P_d) + K_vP_1P_2P_d\right]}{K_vP_1P_2P_ds} \qquad \text{(IV-19)}$$

From this expression open-loop zeros and poles may be obtained and the locus plotted as before. However, the locus will represent the paths in the s-plane which the closed-loop roots travel as the zero of the lag dipole, Z_d, is varied. As Z_d is varied from zero to infinity, the closed-loop roots will move from the open-loop poles to the open-loop zeros of $KG'(s)$.

To continue this illustration numerically, the constants in $KG'(s)$

are given values from the example of Figure IV-6 as follows:

$K_v = 1.48$

$P_1 = +2 + j2$

$P_2 = +2 - j2$

$P_d = +0.5$

Substitution of these values into Equation (IV-19) yields

$$KG'(s) = \frac{Z_d}{5.92} \cdot \frac{s(s+2+j2)(s+2-j2)(s+0.5)+5.92}{s} \qquad \text{(IV-20)}$$

Multiplying out the numerator of KG'(s) and factoring for the approximate open-loop zeros gives

$$KG'(s) = \frac{Z_d}{5.92} \cdot$$

$$\cdot \frac{(s+0.06+j0.83)(s+0.06-j0.83)(s+2.19+j2.0)(s+2.19-j2)}{s} \qquad \text{(IV-21)}$$

The root locus plot of KG'(s) is given in Figure IV-7. Although the open-loop function is strange, in that it possesses four open-loop zeros and only one open-loop pole, the rules for plotting the locus listed in Chapter II may be followed without exception. Since there are three more zeros than poles, three of the locus branches originate at poles at infinity. The asymptotes of these branches approaching from infinity may be found as before. Four open-loop zeros require four root locus branches and four closed-loop roots.

The position of a closed-loop root on a locus branch is determined by a definite value of the variable zero of the lag dipole, Z_d. For purposes of design, one can select a desirable position for a root and handily evaluate Z_d with a spirule. He then can immediately determine

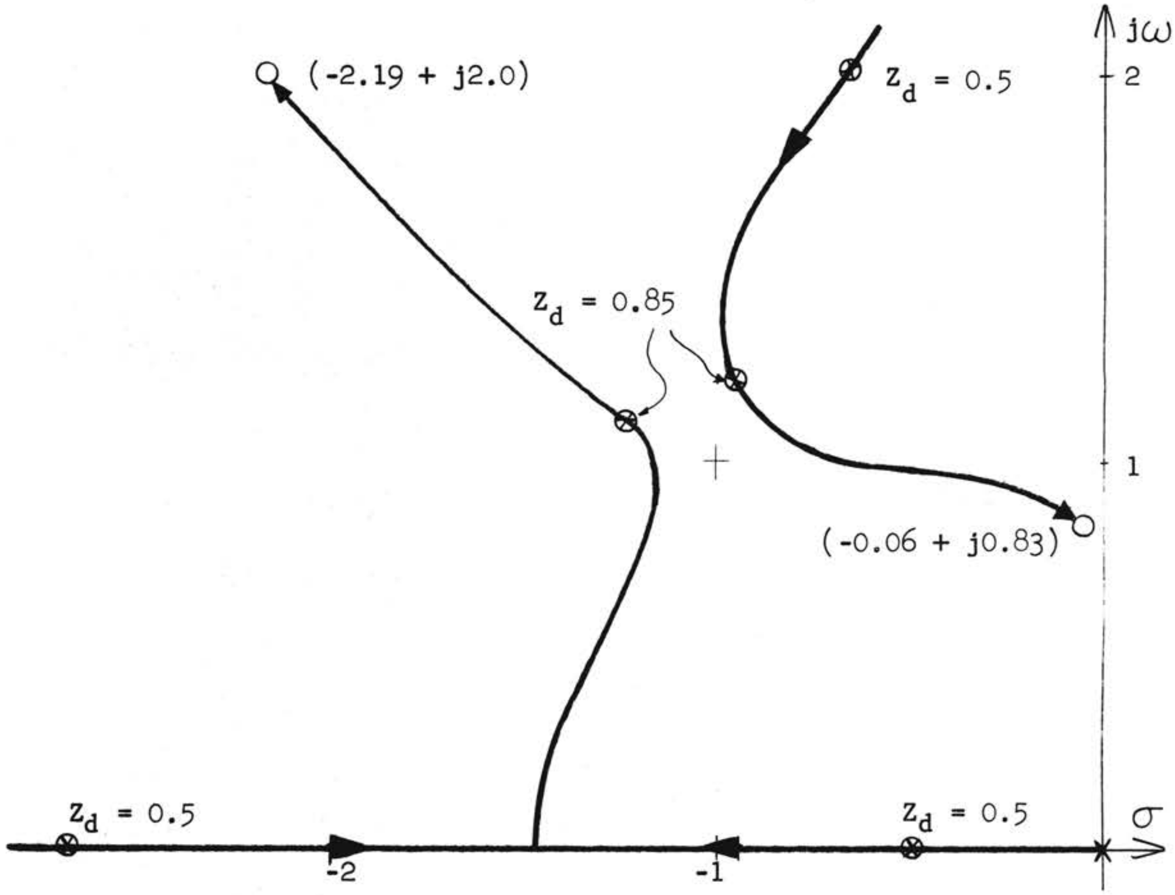


Figure IV-7. Root Locus of $KG'(s) = \frac{Z_d}{5.92} \cdot$

$$\cdot \frac{(s+0.06+j0.83)(s+0.06-j0.83)(s+2.19+j2.0)(s+2.19-j2.0)}{s}$$

the positions of the remaining closed-loop roots on the other locus branches. Should one of these root locations be undesirable for some reason, a compromise selection of root locations can easily be made.

An interesting comparison can be made between the locus of Figure IV-7 and the paths of the closed-loop roots of Figure IV-6. These paths are indicated in Figure IV-6 by dotted lines connecting the roots. For a fixed value of velocity gain constant, $K_v = 1.48$, all the information furnished by the root locus family in Figure IV-6 is presented more fully in Figure IV-7. Even if the gain were available for variation over a certain range, the technique of single parameter variation could be applied twice with the upper and lower limits of gain being used.

The closed-loop roots for two values of Z_d have been indicated. When $Z_d = 0.85$, the closed-loop roots will appear near $s = -1 + j$. This particular value of Z_d would improve system stability without altering the steady-state error of the uncompensated system.

Another interesting result is manifested when $Z_d = 0.5$, causing the pole-zero pair of the dipole to coincide. The effect of this coincidence is not to completely nullify the action of the dipole. Rather, the original roots of the uncompensated system are produced, plus one real root at the dipole position $s = -0.5$.

The single parameter variation method just illustrated could be employed with the pole of the dipole compensator as the variable rather than the zero. For that matter, any single real pole or zero of an open-loop function can be designated as the variable in a root locus plot.

Double Parameter Variation

An extension of the single parameter variation technique allows the simultaneous variation of any real pole and any real zero of the open-loop function, provided the two elements are linked by a constant. The relation of the pole and zero through a constant may be given in two forms. For instance, the pole-zero pair of a dipole compensator may be varied while maintaining fixed separation. In this case, the expression for the dipole is

$$G(s) = \frac{s + P_d + D}{s + P_d} \qquad \text{(IV-22)}$$

where the value of the dipole zero, Z_d, has been replaced by the sum $(P_d + D)$, D being the fixed distance of separation. D can assume positive or negative values depending on the use of a lag or lead dipole.

The second type of relationship between the pole-zero pair is determined by a multiplicative constant. If for every dipole position the pole and zero are related as

$$Z_d = mP_d \tag{IV-23}$$

then the dipole can be expressed as

$$G(s) = \frac{s + mP_d}{s + P_d} \tag{IV-24}$$

where the constant (m) can be greater or less than one, depending on whether a lag or lead dipole is intended. Here the pole-zero separation of the dipole will change as P_d varies.

Equations (IV-22) and (IV-24) involve only one variable, P_d, and the movement of the entire dipole along the negative real axis is described through its variation. Obviously, the elimination of Z_d from the dipole expressions is arbitrary and if desired, Equations (IV-22) and (IV-24) may be written with Z_d as the variable.

Employing P_d as the variable implies that the root locus plot obtained will be the locus of the closed-loop roots in the s-plane as a function of P_d. As in single parameter variation, a new open-loop function must be derived with P_d positioned in the numerator of the function and segregated from the polynomial in (s).

For purposes of further illustration of this technique, two derived open-loop functions will be determined employing the fixed separation dipole of Equation (IV-22) and the variable separation dipole of Equation

(IV-24).

Assume that a unity feedback control system is to undergo series lead or lag compensation. The compensated open-loop transfer function is of the form

$$KG_c(s) = \frac{K(s + Z_1)}{s(s + P_1)(s + P_2)} \cdot \frac{(s + P_d + D)}{(s + P_d)} \tag{IV-25}$$

The gain constant, K, is to remain at a fixed value as the dipole is varied. Note that the velocity gain constant, K_v, must change in value with variation of the dipole, since the ratio Z_d/P_d is not a constant when the dipole possesses fixed separation.

The closed-loop function of $KG_c(s)$ becomes

$$\frac{C}{R}(s) = \frac{KG_c(s)}{1 + KG_c(s)} \tag{IV-26}$$

$$= \frac{K(s + Z_1)(s + P_d + D)}{s(s+P_1)(s+P_2)(s+P_d) + K(s+Z_1)(s+P_d+D)} \tag{IV-27}$$

Equating the denominator of Equation (IV-27) to zero yields the characteristic equation

$$s(s+P_1)(s+P_2)(s+P_d) + K(s+Z_1)(s+P_d+D) = 0 \tag{IV-28}$$

which through algebraic manipulation is brought to form as

$$s^4 + (P_1+P_2)s^3 + (P_1P_2+K)s^2 + K(Z_1+D)s + KZ_1D +$$
$$+ P_d\left[s^3 + (P_1+P_2)s^2 + (P_1P_2+K)s + KZ_1\right] = 0 \tag{IV-29}$$

Equation (IV-29) can be written as

$$\left[s^4 + (P_1+P_2)s^3 + (P_1P_2+K)s^2 + K(Z_1+D)s + KZ_1D\right]$$

$$\left[1 + \frac{P_d\left[s^3 + (P_1+P_2)s^2 + (P_1P_2+K)s + KZ_1\right]}{s^4 + (P_1+P_2)s^3 + (P_1P_2+K)s^2 + K(Z_1+D)s + KZ_1D}\right] = 0 \qquad \text{(IV-30)}$$

The second factor of Equation (IV-30) when equated to zero will possess the same roots as does the characteristic equation. Comparison of this factor with the denominator of Equation (IV-26) indicates that the new open-loop function will be analogous to $KG_c(s)$ and may be expressed as

$$KG'(s) = \frac{P_d\left[s^3 + (P_1+P_2)s^2 + (P_1P_2+K)s + KZ_1\right]}{s^4 + (P_1+P_2)s^3 + (P_1P_2+K)s^2 + K(Z_1+D)s + KZ_1D} \qquad \text{(IV-31)}$$

After numerical values of P_1, P_2, Z_1, K and D have been substituted into $KG'(s)$, the numerator and denominator can be factored to obtain the open-loop zeros and poles. Notice that the open-loop zeros in this particular case are a function of gain, K, and not dependent on the dipole separation D. This fact would prove desirable if several plots were to be made with different values of dipole separation and a fixed gain. It would not be necessary to refactor the numerator of $KG'(s)$ each time.

It is interesting to note that P_1 and P_2 are found only in product form (P_1P_2) or sum form (P_1+P_2) in $KG'(s)$. This allows P_1 and P_2 to occur as complex conjugates and still yield real coefficients of (s) in the polynomials of $KG'(s)$.

The second form of pole-zero relationship provides a constant ratio $Z_d/P_d = m$. This follows from Equation (IV-23). The effective gain constant of a system depends upon the ratio of open-loop zeros to the open-loop poles. For a compensated type 1 system of the form

$$KG_c(s) = \frac{K(s+Z_1)}{s(s+P_1)(s+P_2)(s+P_3)} \cdot \frac{s+Z_d}{s+P_d} \tag{IV-32}$$

the velocity gain constant K_v is given by

$$K_v = K \frac{Z_1}{P_1 P_2 P_3} \cdot \frac{Z_d}{P_d} \tag{IV-33}$$

or

$$K_v = K \frac{Z_1}{P_1 P_2 P_3} \cdot m \tag{IV-34}$$

Thus the gain, K, and velocity gain constant, K_v, can both be fixed in value as the dipole is varied, so long as the ratio $Z_d/P_d = m$ remains constant. This property is valuable when velocity-lag error must remain unchanged.

A similar algebraic procedure utilized to derive the open-loop function given by Equation (IV-31) may be applied to Equation (IV-32) after Z_d is replaced with mP_d. The result is

$$KG'(s) = \frac{P_d\left[s^4+(P_1+P_2+P_3)s^3 + (P_1P_2+P_1P_3+P_2P_3)s^2 + (P_1P_2P_3+mK)s + mKZ_1\right]}{s\left[s^4+(P_1+P_2+P_3)s^3 + (P_1P_2+P_1P_3+P_2P_3)s^2+(P_1P_2P_3+K)s + KZ_1\right]} \tag{IV-35}$$

Upon substitution of numerical values and factoring, KG'(s), given by Equation (IV-35), will produce four open-loop zeros and five open-loop poles. The root locus of KG'(s) will describe the movement in the s-plane of the closed-loop roots of the system of Equation (IV-32) with a variation of P_d at a fixed gain. A lead dipole will be applied to the system if (m) is less than one and a lag dipole will be produced by letting (m) be greater than one.

Should m = 1, the pole and zero of the dipole will coincide. In addition, if K = 1, then the open-loop poles of KG'(s) coincide with the open-loop zeros with the exception of the open-loop pole at the origin. In this case, the closed-loop roots would remain fixed with the variation of the dipole, except for one real root which would move out the negative real axis with the dipole (now in "piggyback" condition).

Any two of the three original open-loop poles, P_1, P_2 or P_3 may be complex conjugates. The coefficients of (s) in the polynomials of Equation (IV-35) are such that only real values will occur.

Single and double parameter variation techniques are useful in the selection of appropriate dipole compensators. This is particularly true when steady-state error is not to be changed appreciably. The location of all closed-loop roots for a given dipole configuration are furnished. In addition, the choice of varying a lead or a lag dipole involves only the changing of the value of one constant, D or m.

The only serious disadvantage of this method is the factoring of at least fourth or fifth order polynomials. This operation is required in putting the derived open-loop function in a form amenable to plotting the open-loop poles and zeros. However, in most situations exact factoring is not necessary since the over-all purpose of the locus plot is to display the movement trends of the closed-loop roots. If a particular dipole position seems to produce desirable root locations, then more accurate factoring of the polynomials may be indicated.

The use of computer techniques is indicated here and the parameter variation method presents no serious problems if such facilities are available. In addition, the root locus method itself can be applied directly to the problem of factoring polynomials. (20).

CHAPTER V

SUMMARY

The design of series dipole compensators with the root locus technique depends upon the knowledge of the location of the closed-loop roots for a given dipole selection. By virtue of this information, which can be derived directly from the root locus plot, performance characteristics of the system can be predicted.

The purpose of this thesis is to investigate the methods for exploiting the root locus technique for compensator design. The common feature of the methods examined is that the movement of all of the closed-loop roots in the complex plane versus dipole parameter variation is displayed. Trends of migration of the roots are usually apparent with only a few trial dipole positions. This allows quick rejection of certain dipole configurations which do not yield acceptable compensation.

Fundamentally, two separate techniques are employed with the second of these being presented in two forms. The first method involves the root locus family and yields the greatest amount of information about the compensated system, but, by the same token, requires the greatest amount of time and effort. Use of computing facilities effectively nullifies this disadvantage.

The signal advantage of the root locus family is that system gain remains available as a variable for the design of the compensated system. When steady-state error is not fixed by system specifications, gain variation in conjunction with dipole position normally allows a wide latitude of choice for the closed-loop root locations. Lead or lag dipoles with fixed or variable pole-zero separation may be selected for use in construction of the root locus family.

The second technique employed in the present investigation involves parameter variation. Rather than plotting the position of the closed-loop roots as a function of system gain, the locus is constructed with an element of the dipole as the variable. Additionally, both the pole and zero of the dipole are related by a constant and the pole and zero become the variables. The constant relationship may enforce the condition of fixed separation or variable separation on the pole-zero pair.

The application of parameter variation requires the derivation of a corrected open-loop function from the original function and the subsequent factoring of its numerator and denominator. This constitutes the only additional labor demanded by the parameter variation technique and can be relieved through the use of a computer. The worth of additional information yielded by the plot in terms of dipole compensator design normally repays this extra effort.

BIBLIOGRAPHY

1. Hazen, H. L., "Theory of Servomechanisms." Journal of the Franklin Institute, Vol. 218, September, 1934; pp. 279-331.

2. Hazen, H. L., "Design and Test of a High-Performance Servomechanism." Journal of the Franklin Institute, Vol. 218, November, 1934; pp. 543-580.

3. Evans, W. R., "Graphical Analysis of Control Systems." AIEE Transactions, Vol. 67, 1948; pp. 547-551.

4. Evans, W. R., "Control System Synthesis by the Root Locus Method." AIEE Transactions, Vol. 69, 1950; pp. 66-69.

5. Evans, W. R., Control-System Dynamics. New York: McGraw-Hill Book Company, Inc., 1954.

6. Ross, E. R., T. C. Warren and G. J. Thaler, "Design of Servo Compensation." AIEE Transactions, Vol. 79, 1960, pp. 272-277.

7. Truxal, John G., Automatic Feedback Control System Synthesis. New York: McGraw-Hill Book Company, Inc., 1955, pp. 221-277.

8. Chestnut, Harold, and R. W. Mayer, Servomechanisms and Regulating System Design. Vol. 1., 2d. ed. New York: John Wiley and Sons, Inc., 1959, pp. 161-178.

9. Thaler, George J., and R. G. Brown, Analysis and Design of Feedback Control Systems. 2d. ed. New York: McGraw-Hill Book Company, Inc., 1960, pp. 176-197.

10. Truxal, John G., op. cit.

11. Chestnut, Harold, and R. W. Mayer, op. cit.

12. Truxal, John G., op. cit. p. 230.

13. Evans, Walter R., Control-System Dynamics. New York: McGraw-Hill Book Company, Inc., 1954, pp. 237-241.

14. Chestnut, Harold, and R. W. Mayer, op. cit. pp. 424-430.

15. Truxal, John G., op. cit. p. 259.

16. Chestnut, Harold, and R. W. Mayer, op. cit. pp. 384-387.

17. Chestnut, Harold, and R. W. Mayer, op. cit. pp. 389-391.

18. Smith, Otto J. M., Feedback Control Systems. New York: McGraw-Hill Book Company, Inc., 1958, pp. 275-276.

19. Truxal, John G., op. cit. pp. 272-274.

20. Truxal, John G., op. cit. pp. 266-272.

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